Why are there so many power laws in economics?

Linz, October 2023

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Motivation

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- These things repeatedly pop up in my research
  - Wealth distributions, citations & public recognition of academics, market power (e.g. technology adoption)

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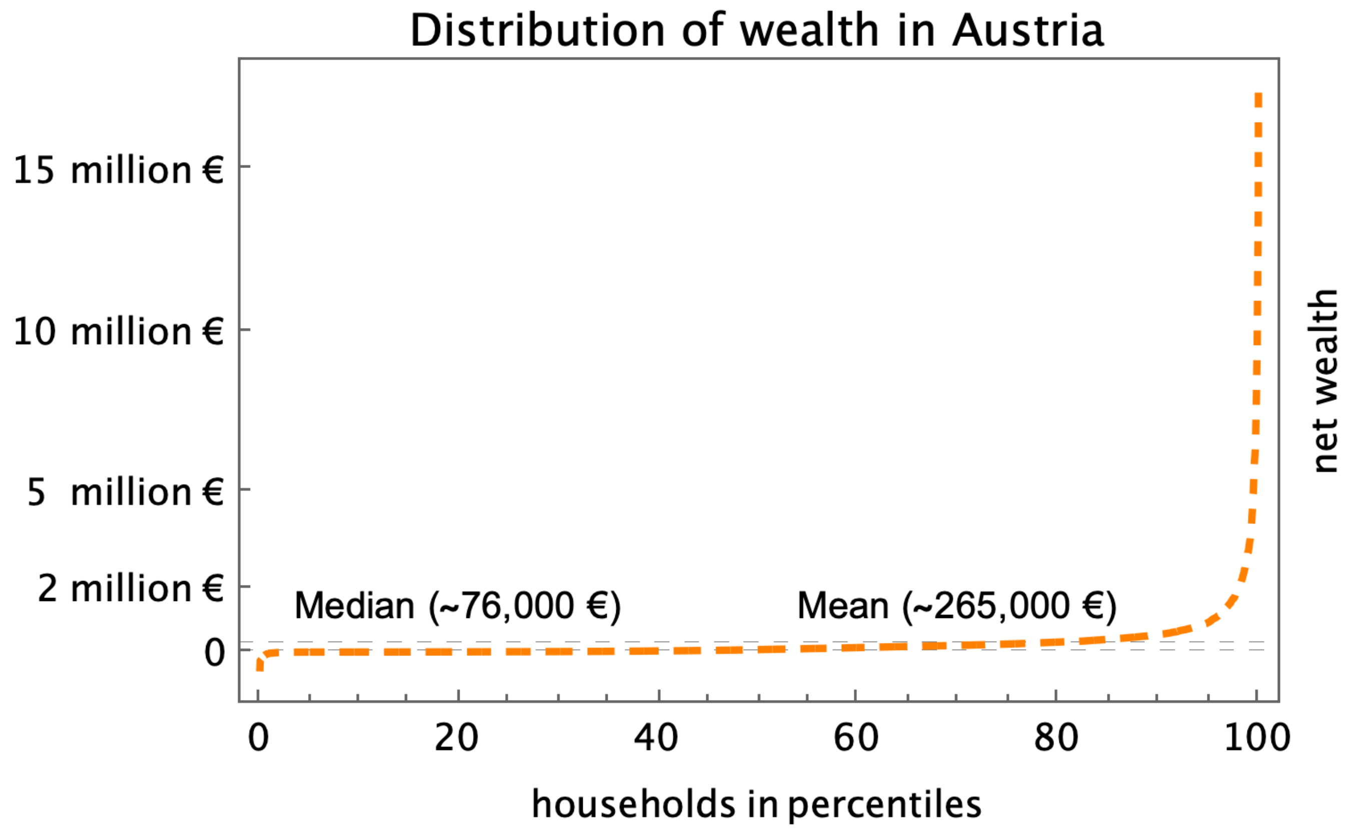

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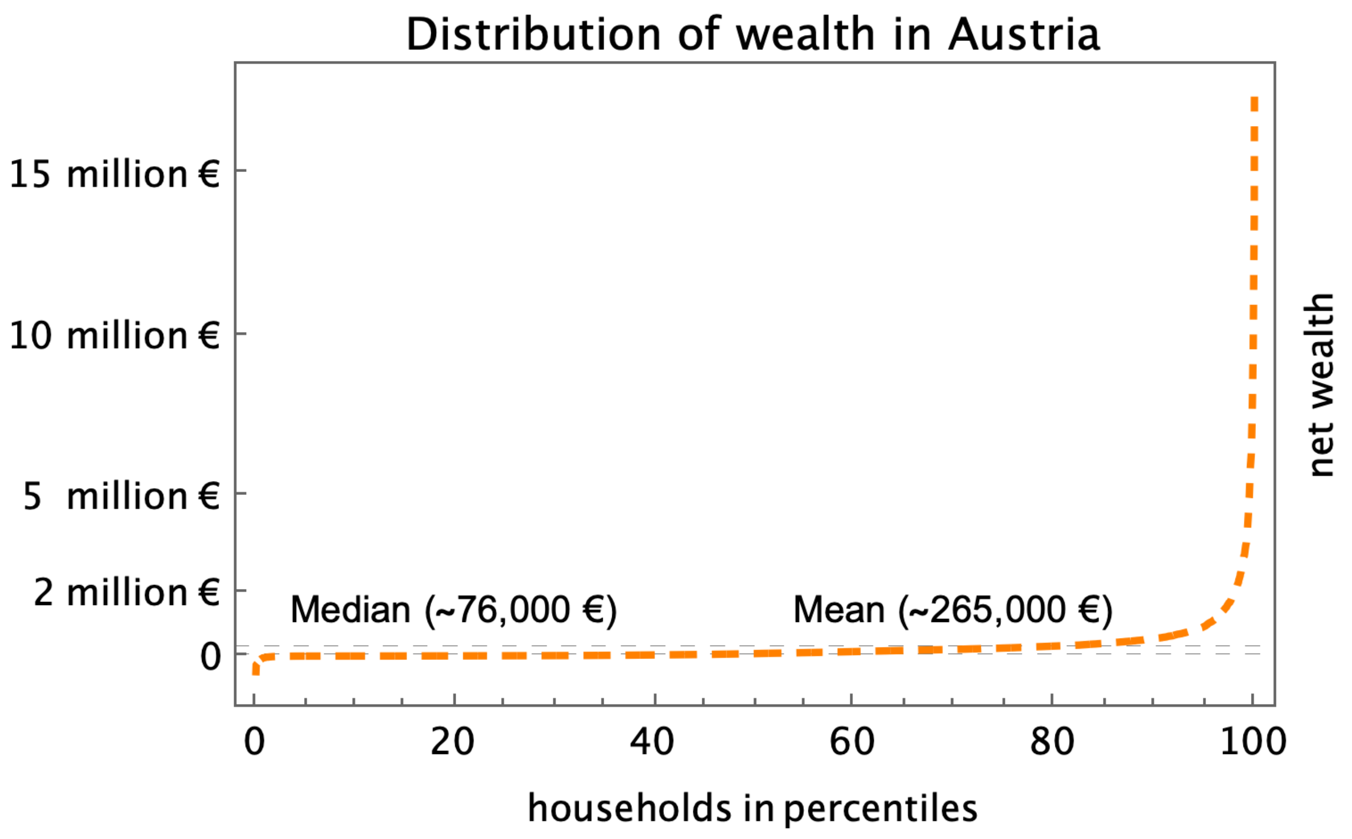

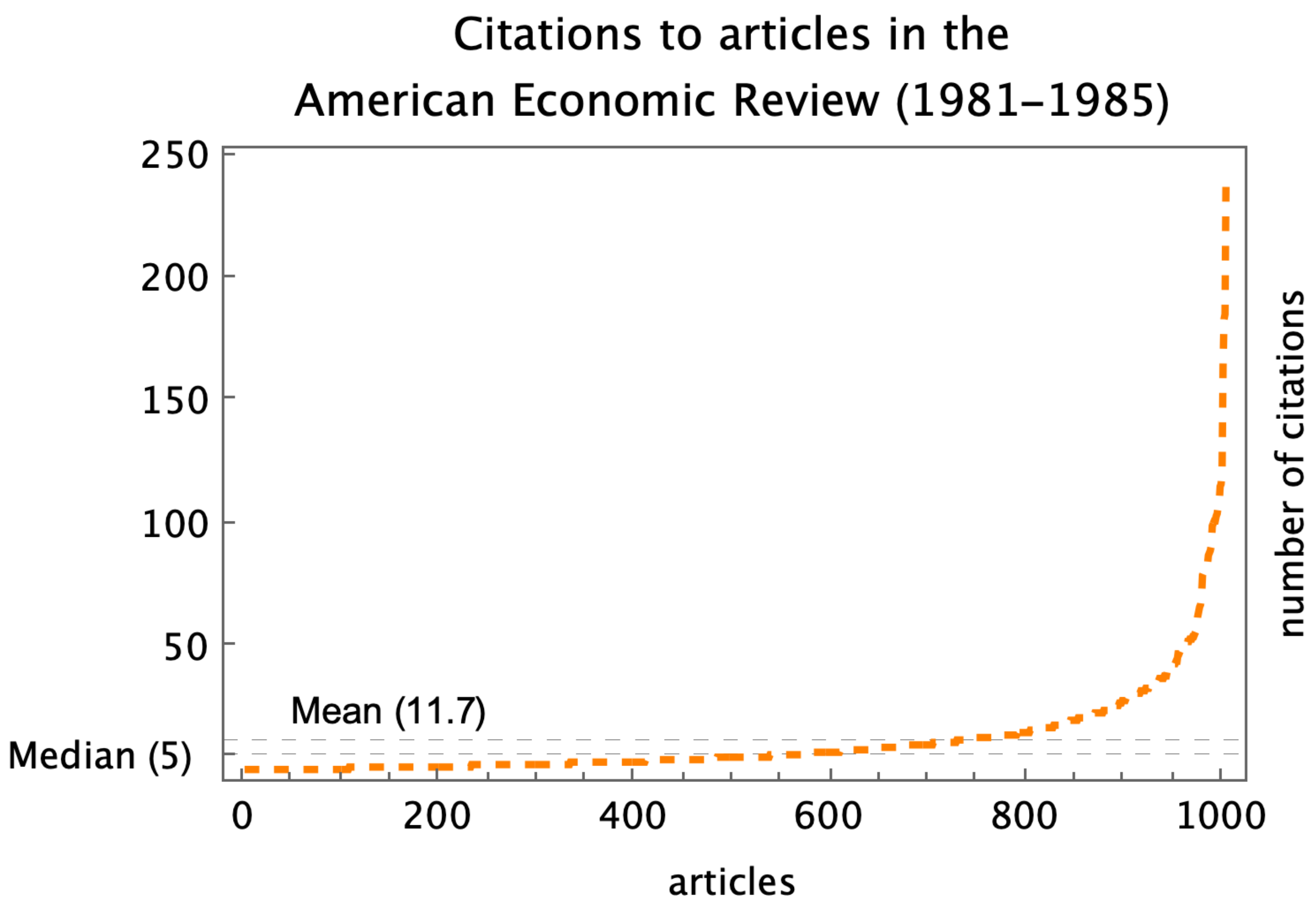

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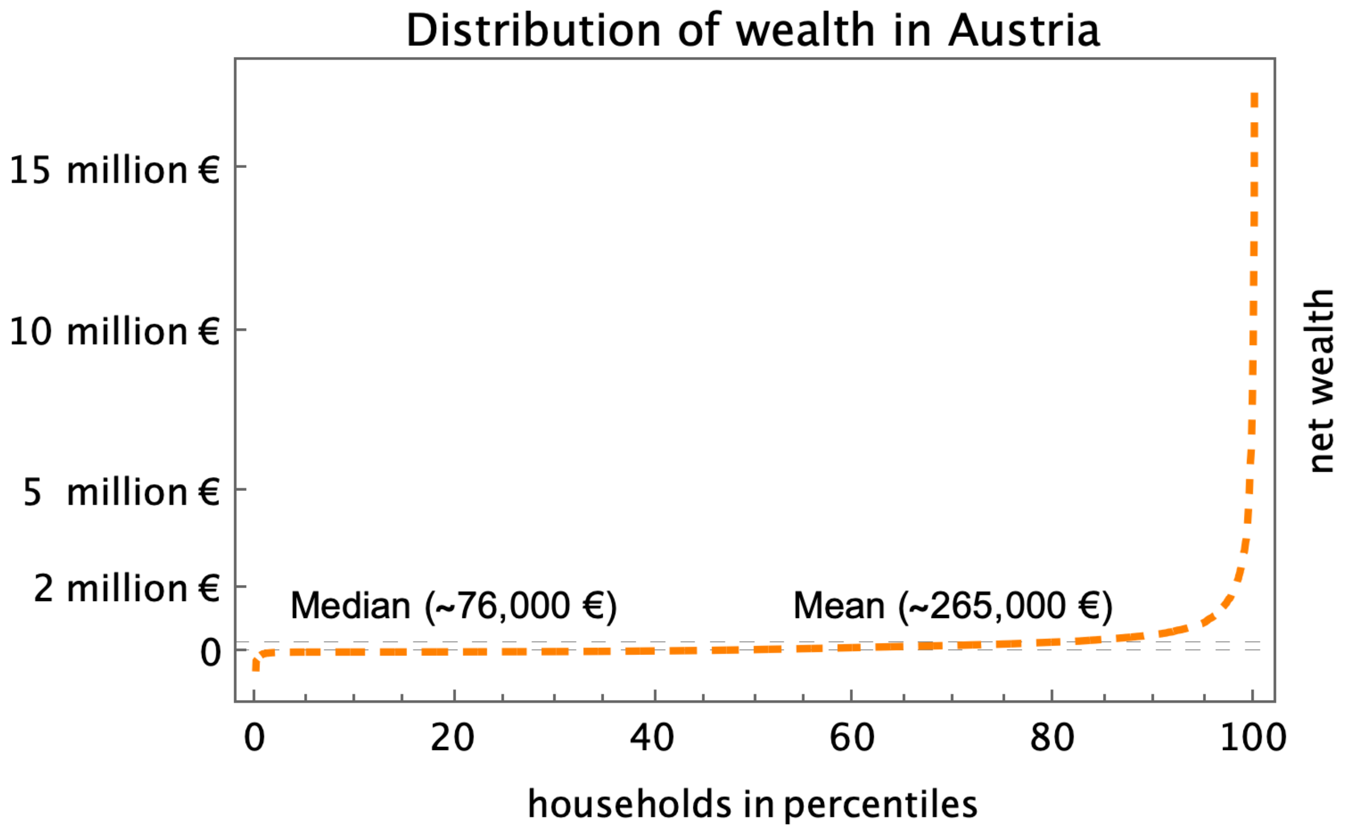

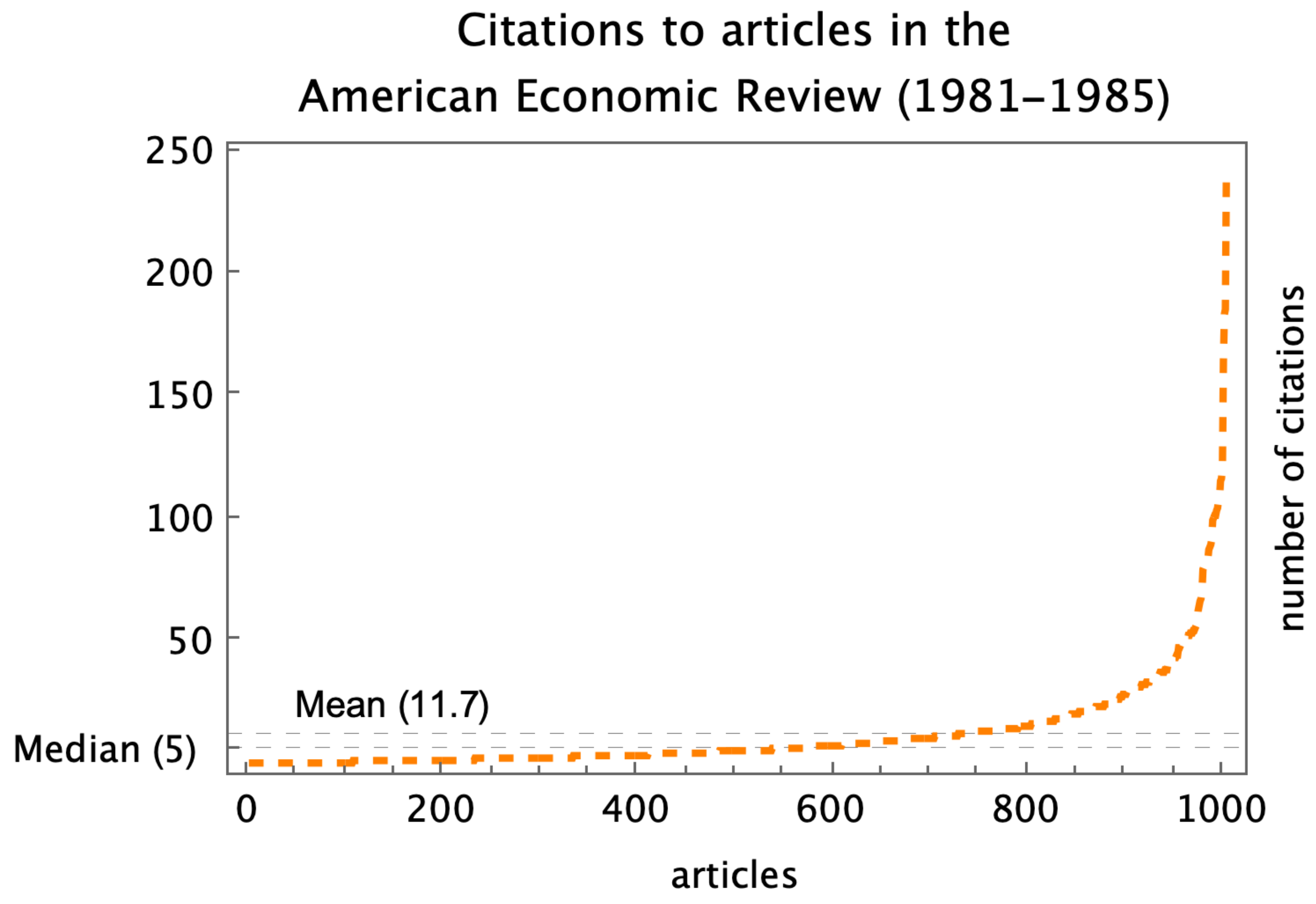

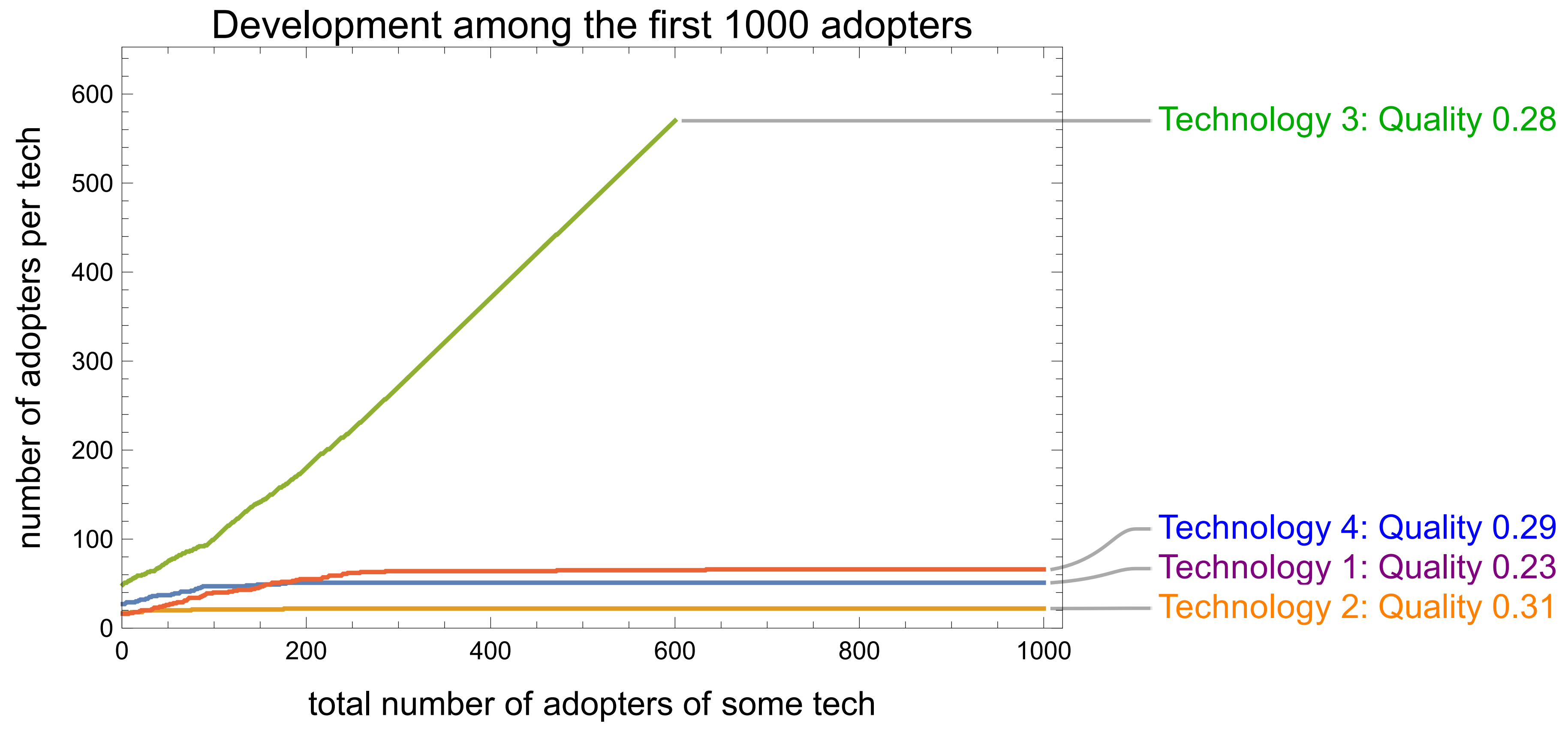

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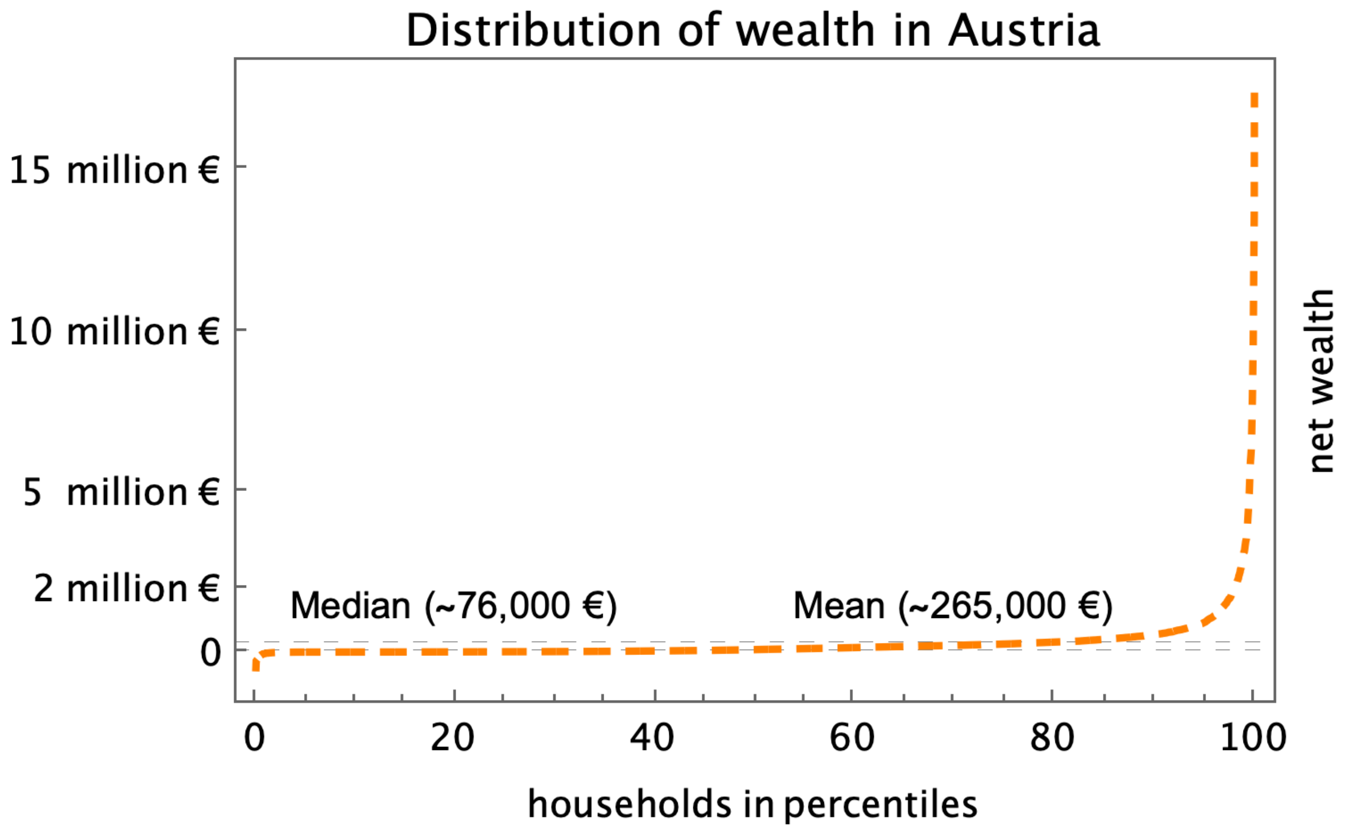
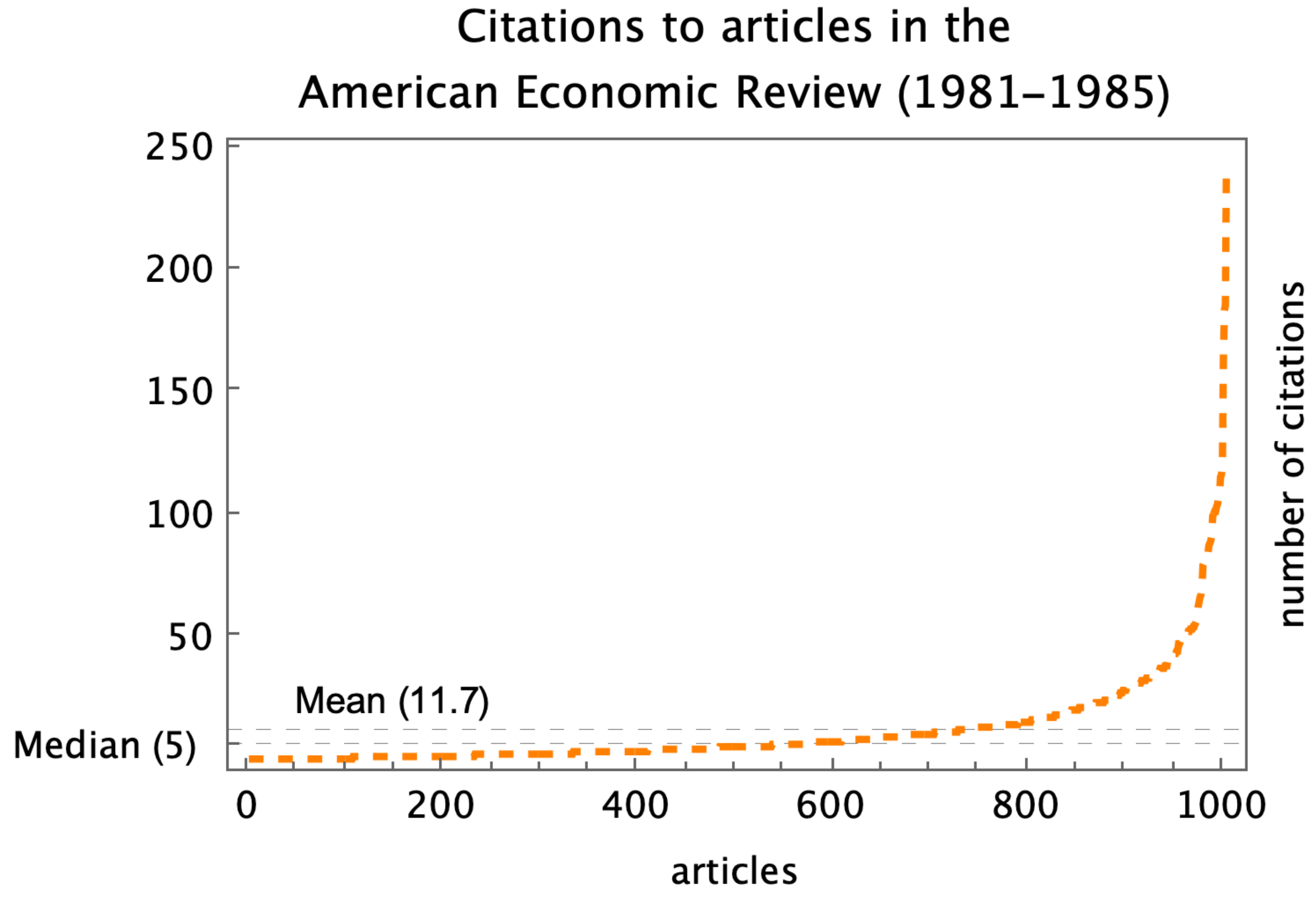
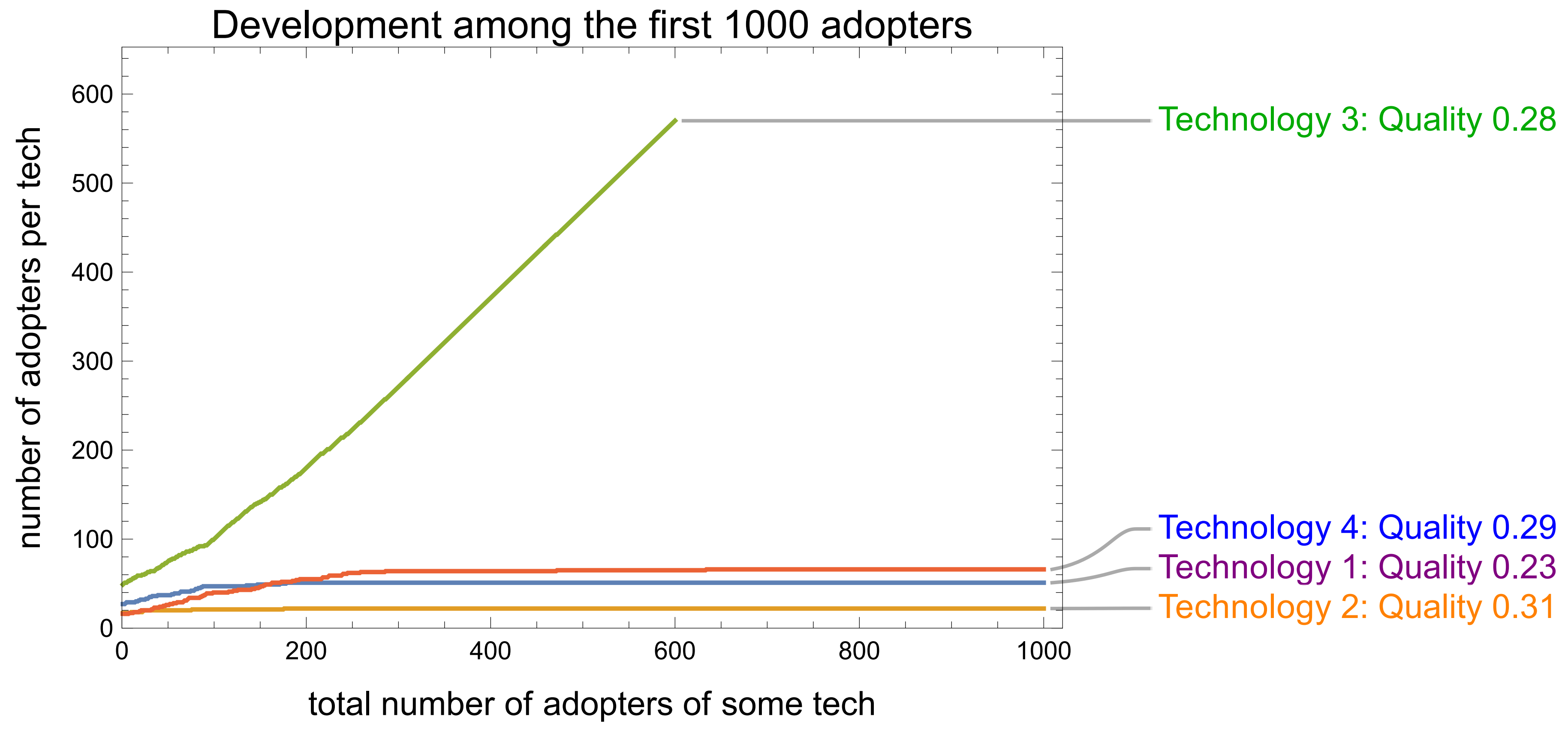




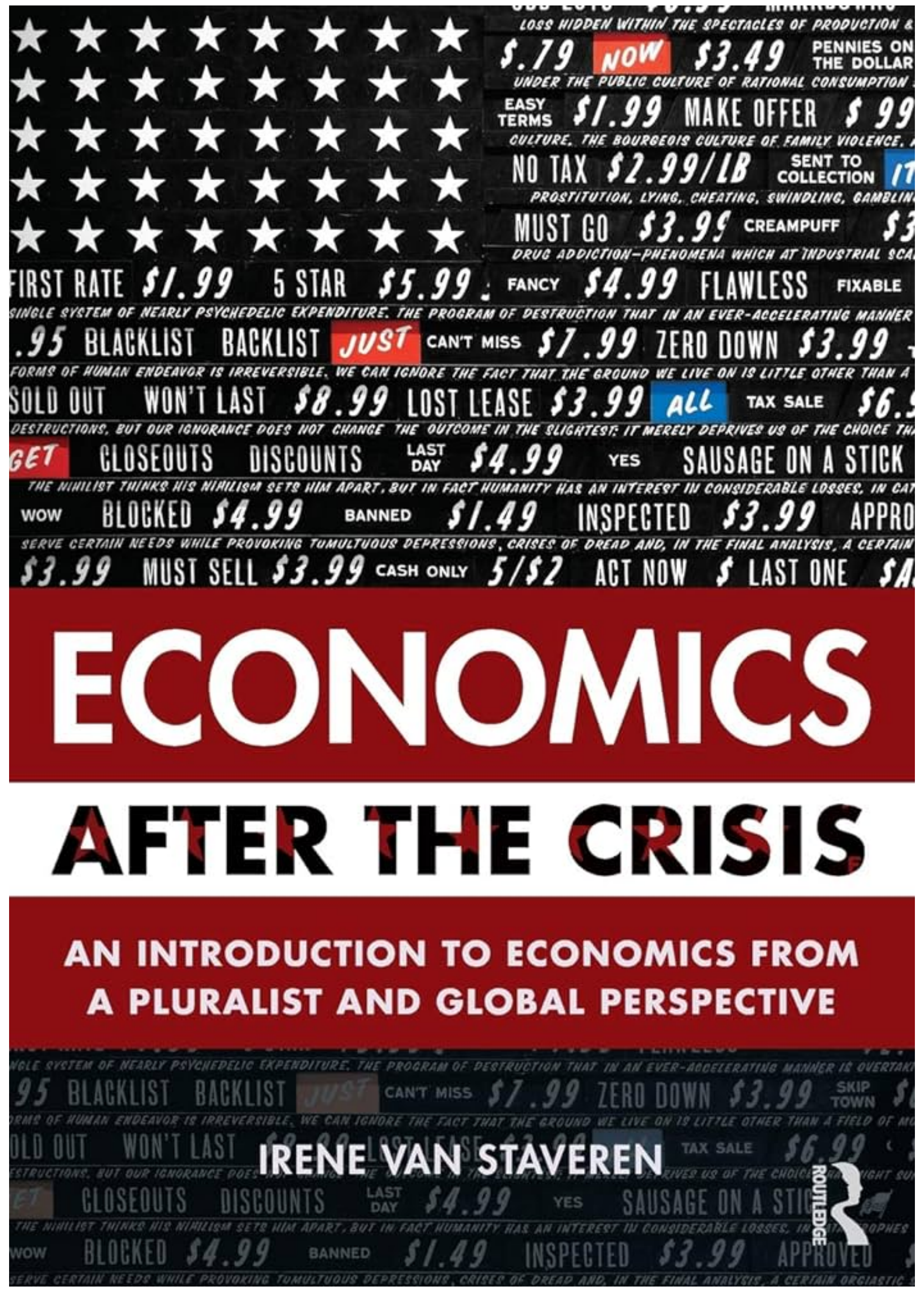


  - They also are found in other phenomena of interest to socio-economists, like city size, firm size, capital income, components of private wealth, the popularity of websites or books, the size of individual social networks etc.

- Power laws are a well-established concept in other sciences (e.g. physics, maths, biology…)
  - Possible starting point for interdisciplinary collaborations – diverse contributions on ‚generative mechanisms'

Why power laws?

- These things repeatedly pop up in my teaching
  - Recent task: Create a new introduction to economics from a pluralist view point for future teachers – „Economic Thought“
  - Stealing from this great book the idea to introduce main economic actors (households, firms, state, etc.) and to provide a first conceptualization to assure that students' basic impressions are aligned – but how to best introduce core economic properties of such actors?
  - My answer: Stratification of actors as a core dimension of interest – info on aggregate + parts!



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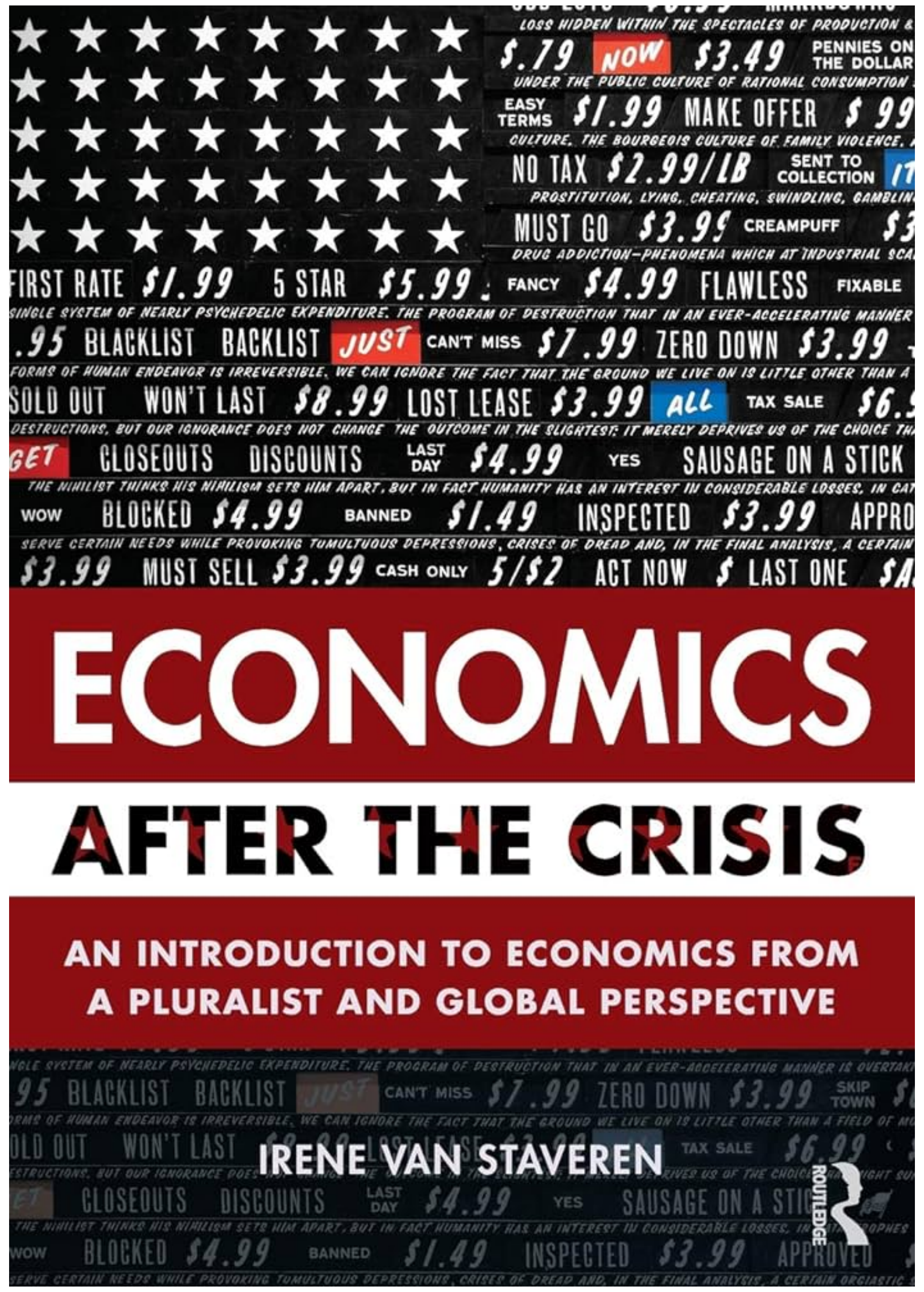

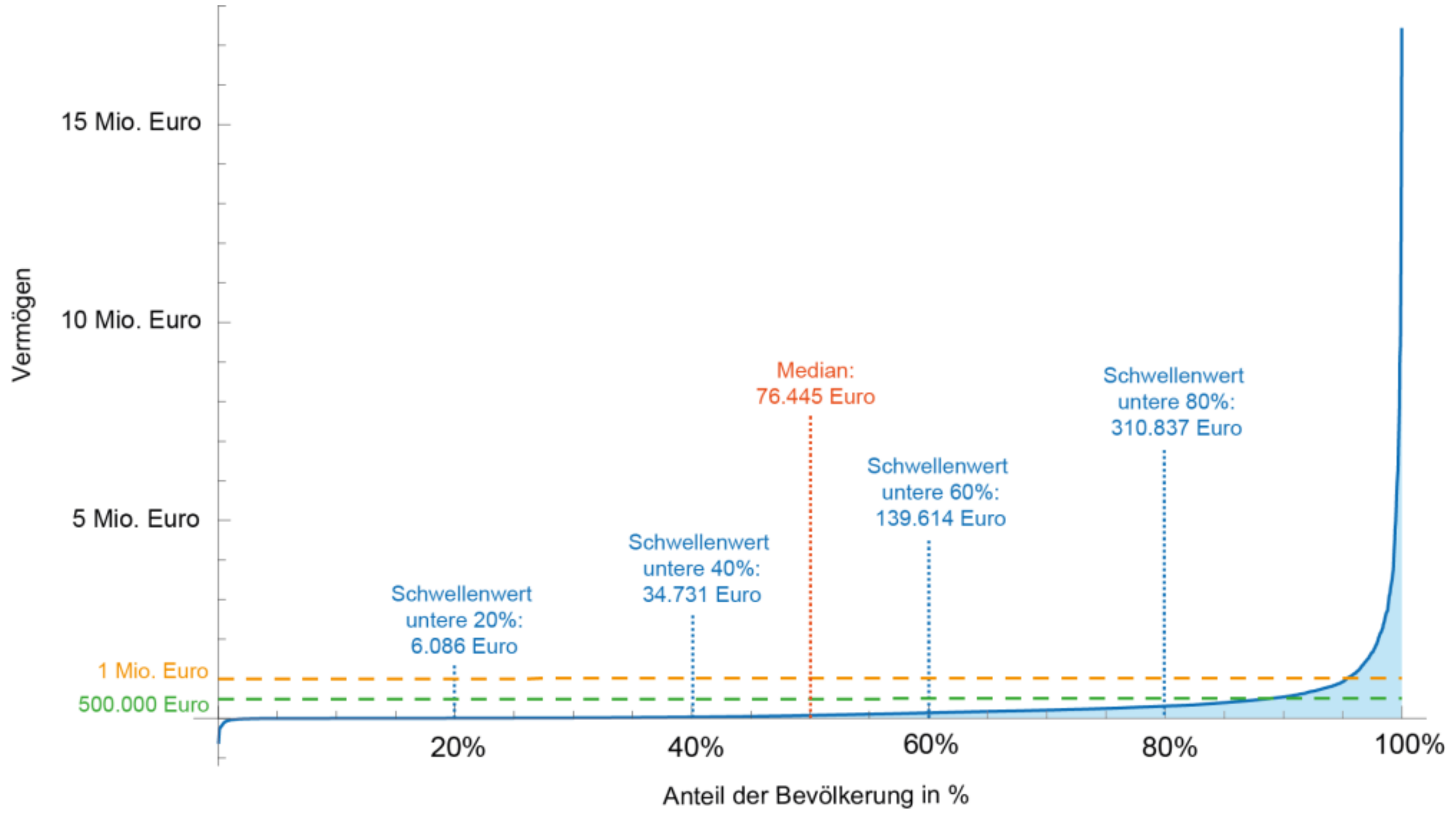

Wealth in Austria (HFCS, Wave I): http://media.arbeiterkammer.at/PDF/MWuG_Ausgabe_122.pdf

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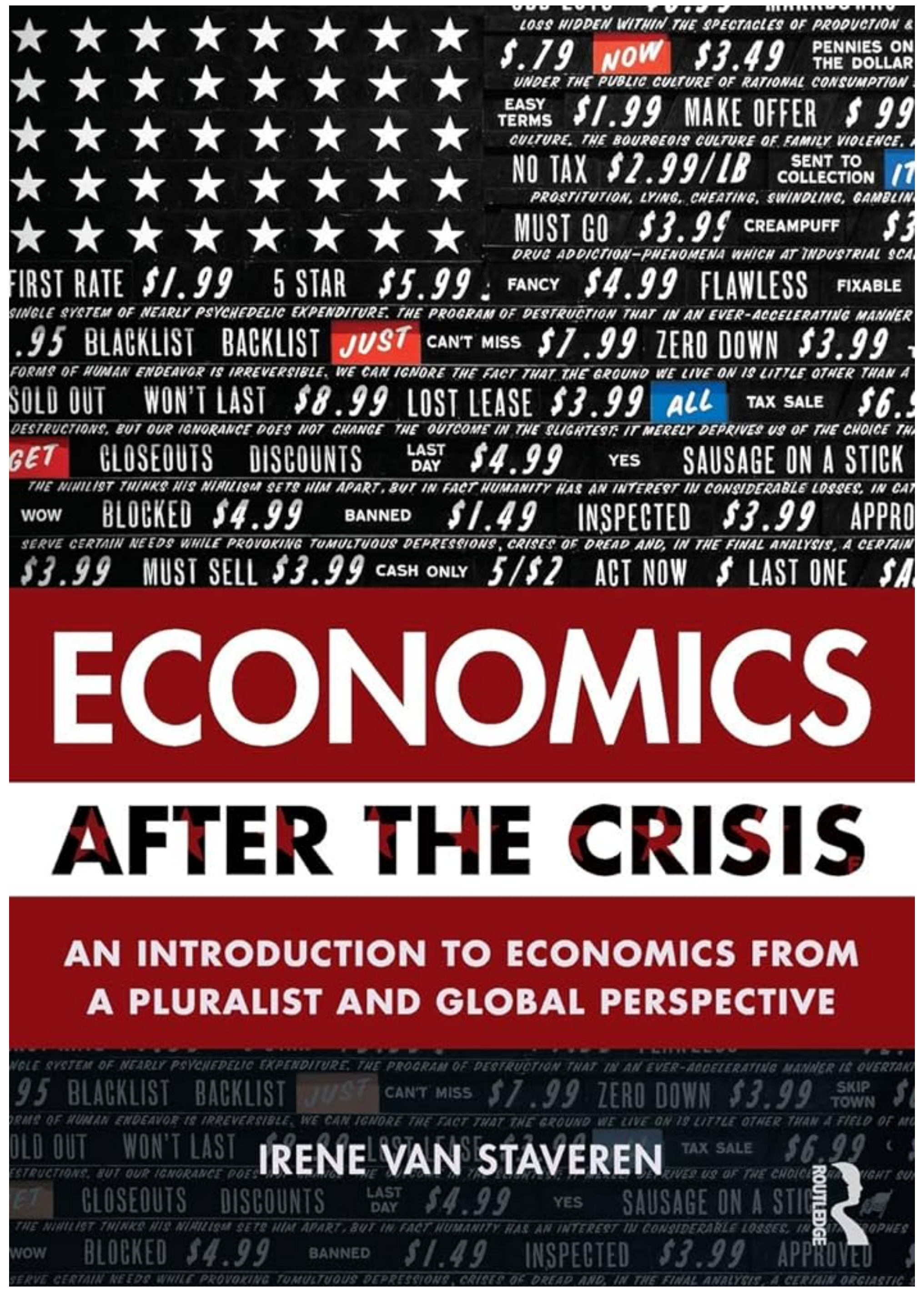

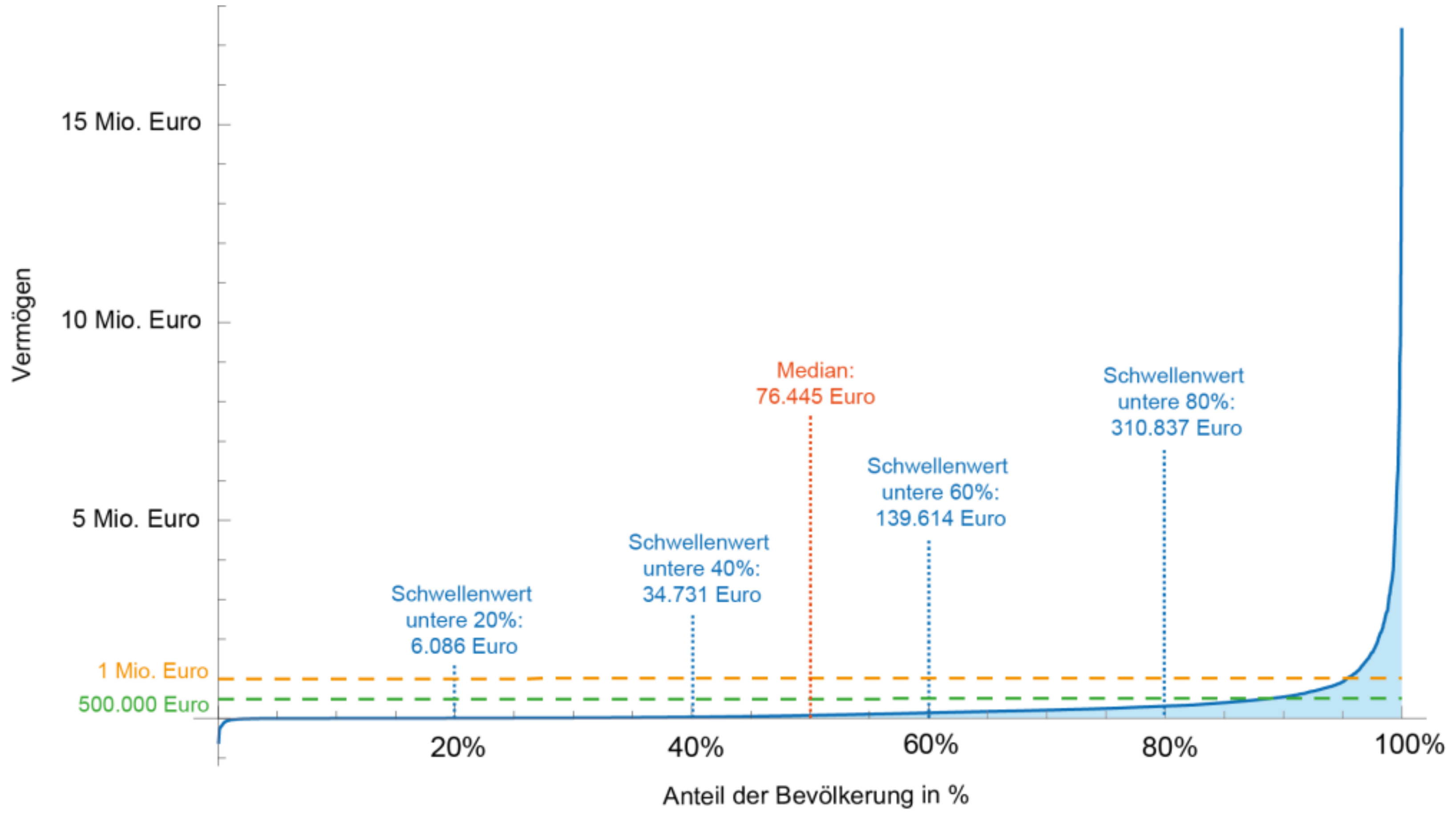

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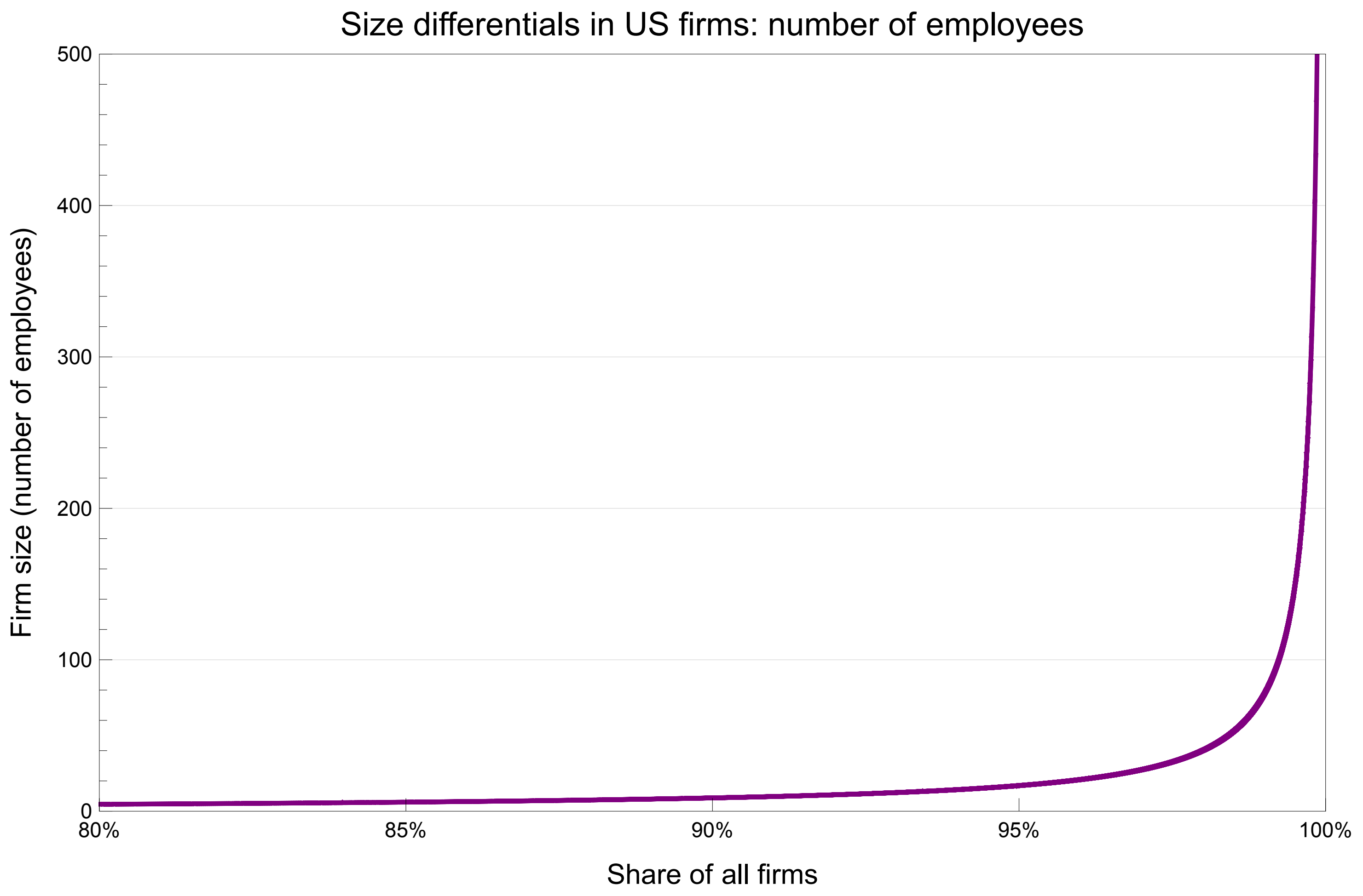


Based on: Axtell, Robert (2001): Zipf Distribution of U.S. Firm Sizes. Science, Vol. 293

Agenda

- Motivation

- Conceptualizing power laws
  - Defining power laws: some variants
  - Talking about power laws: some (preliminary) lessons

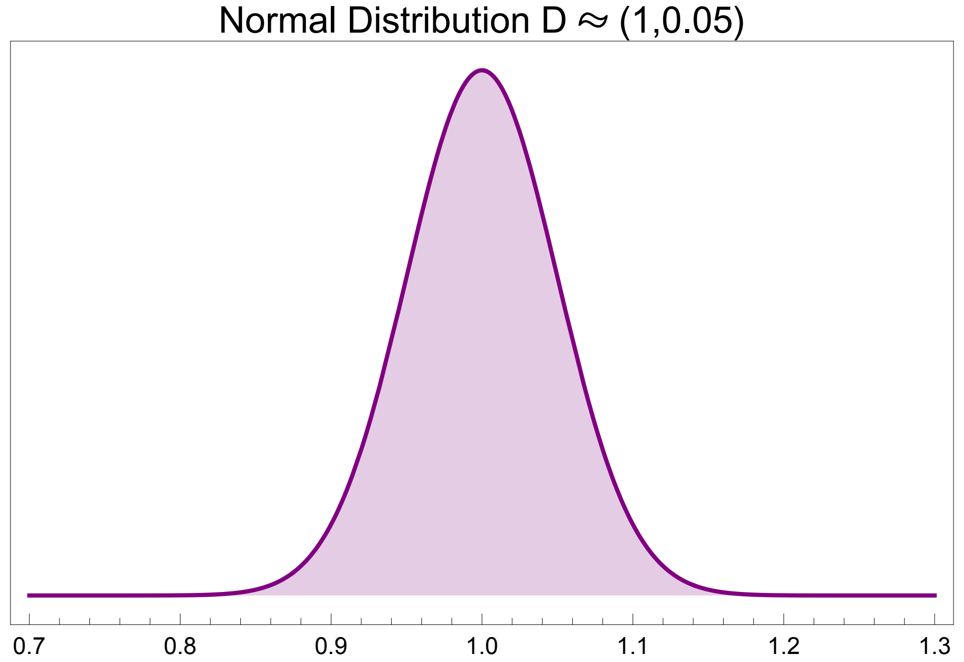
- Generating power laws
  - General overview on generative mechanisms
  - Simple models of cumulative advantage (and the welfare state)
  - Simple models of multiplicative random growth (and the welfare state)
  - Concluding thoughts

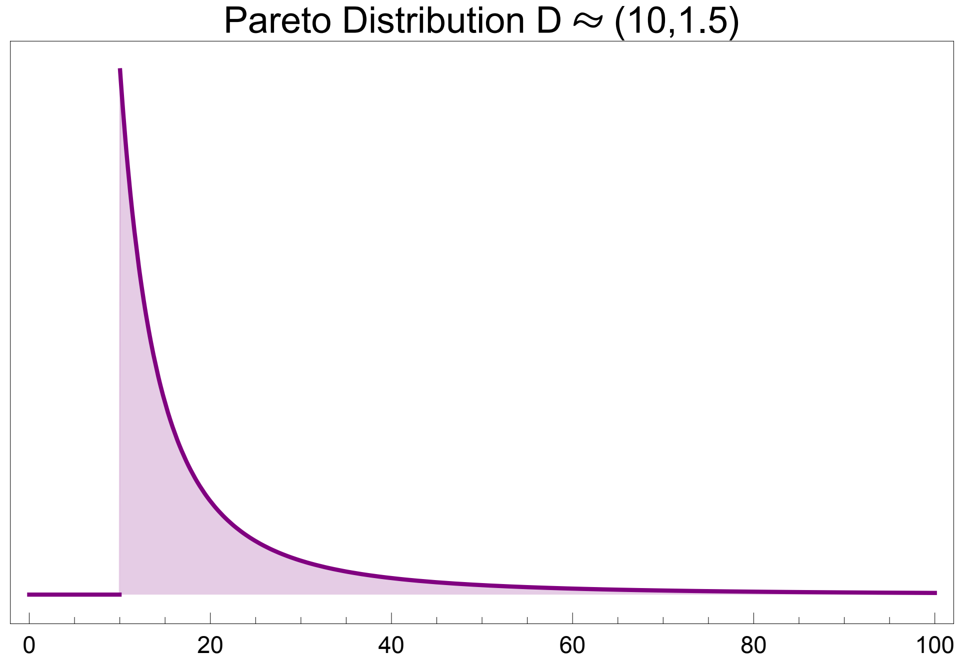
Defining power laws

Power laws as scaling relationships

- Basic functional form: $f(x) = a \cdot x^b$ with $a, b \in \mathbb{R}$
  - When applied to two variables it expresses a scaling relationship (econ-speak: „homogenous of degree *k*“).
  - Biology: „doubling the size of mammals will increase their weight 8-fold“: $weight(size) = a \cdot size^3$
  - Mathematics: „doubling the length of square, will increase its area 4-fold“: $area(length) = a \cdot length^2$
  - Economics: „doubling the number of inputs, will increase output *k*-fold“: $output(inputs) = a \cdot inputs^k$

- When some variable „follows a power law“… $f_X(x) = a \cdot x^b$ with $b < 0$ → The probability of observing x, becomes smaller when x becomes larger, but never truly zero
  - … we typically apply exactly the same reasoning (a scaling relationship) to the variable of interest in absolute and relative terms = a functional relation between absolute value and relative position of the same variable $rank(value) = a \cdot value^b$
  - Caveat: In most cases the power law distribution only applies to some upper segment of the data: $x_i > x_{min}$
  - Probability density functions (PDFs) to illustrate what this means.





Power laws are often ‚discovered' in the history of the sciences

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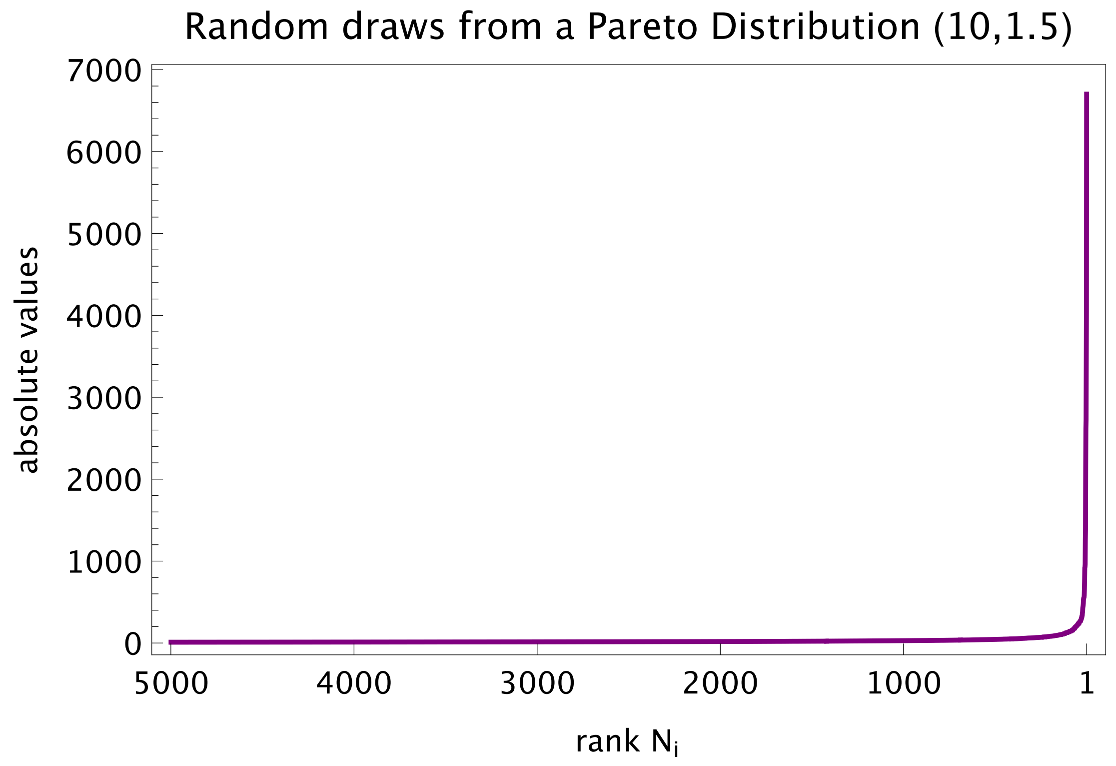

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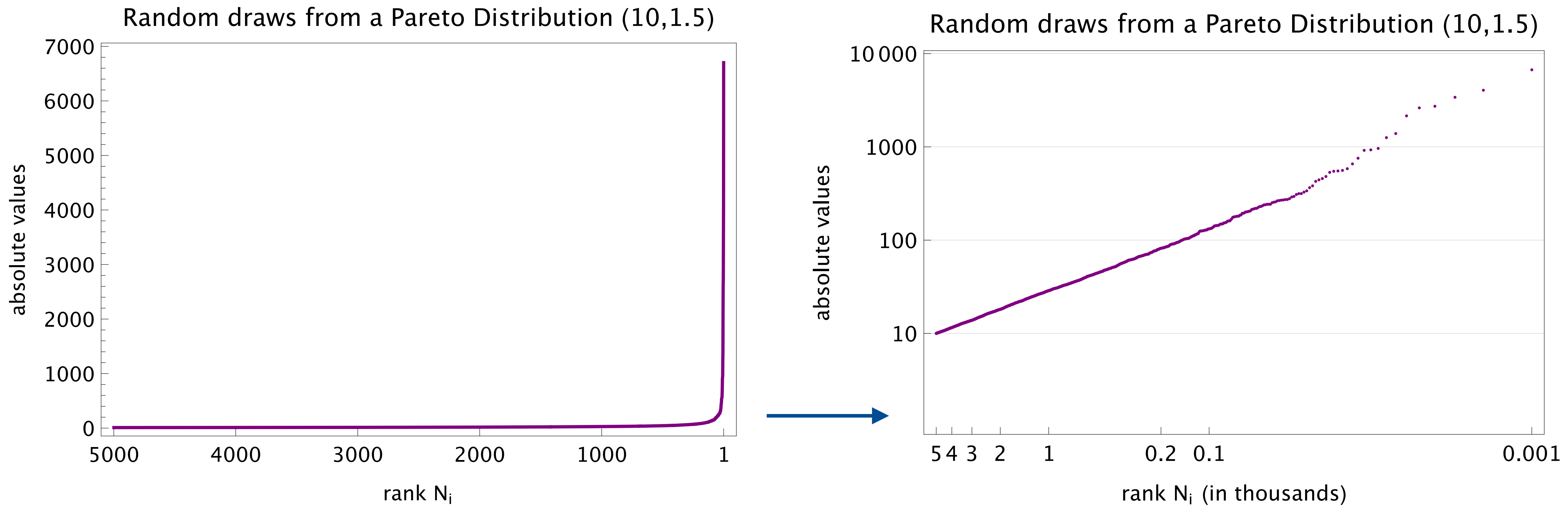

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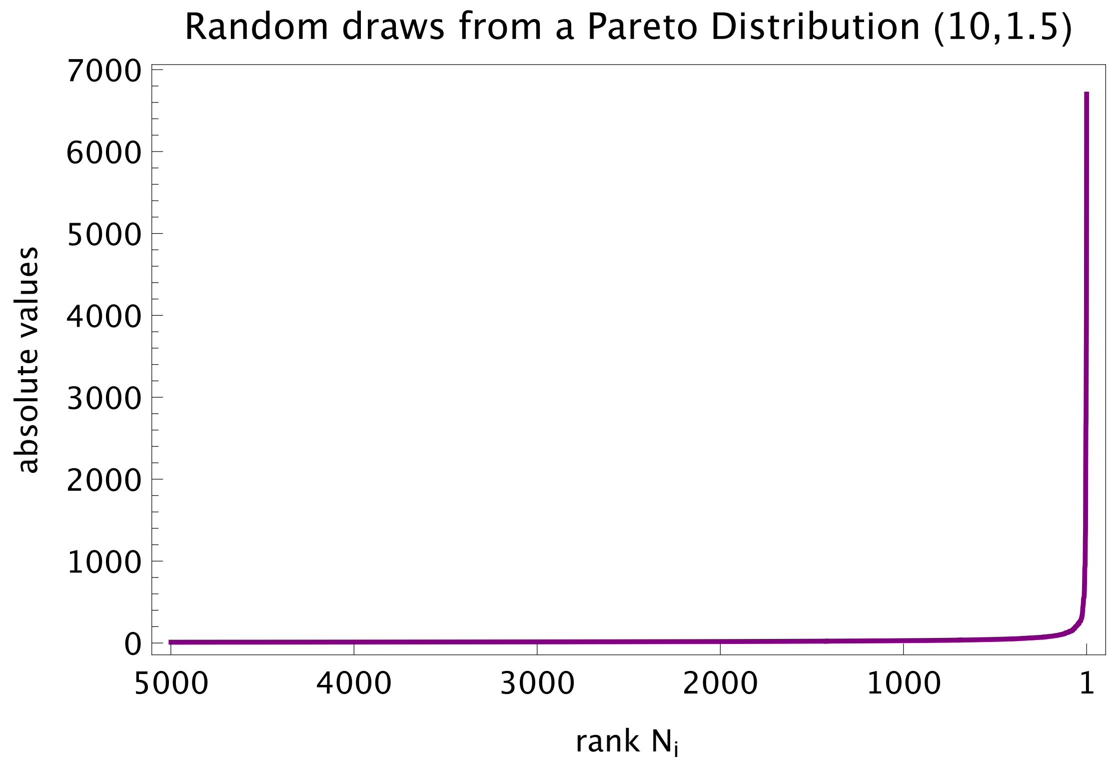

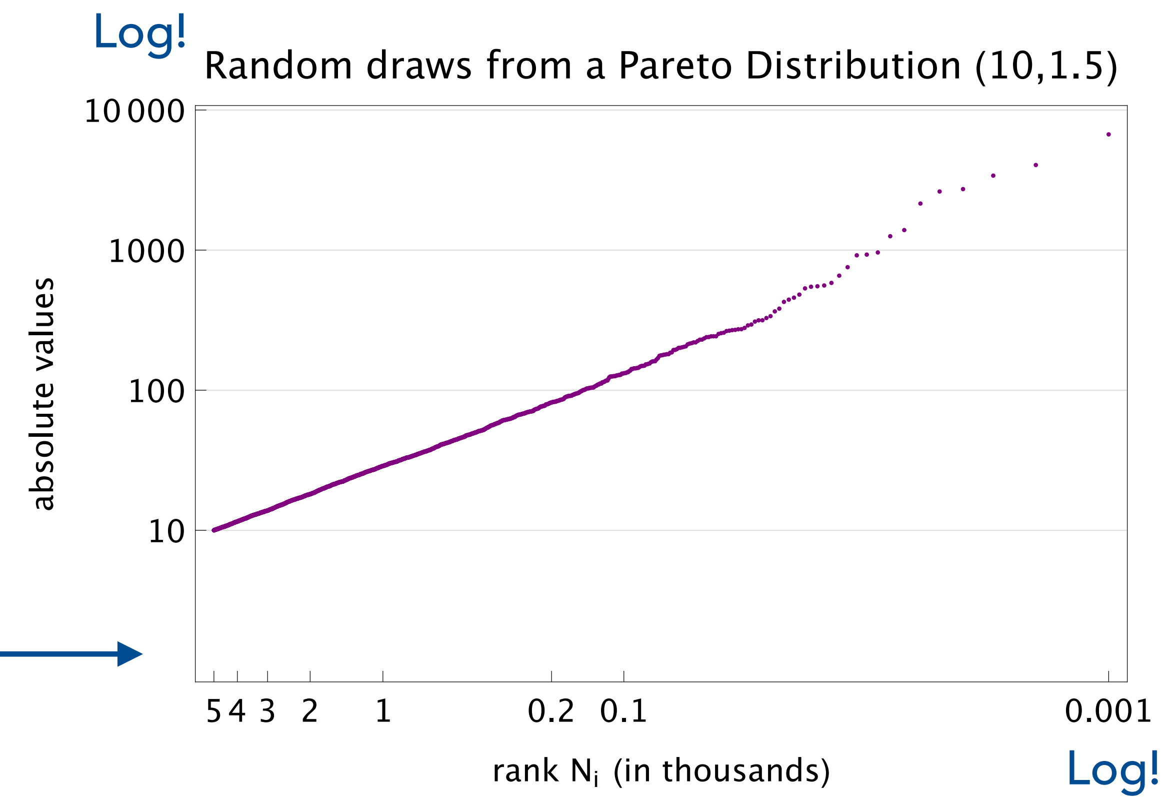

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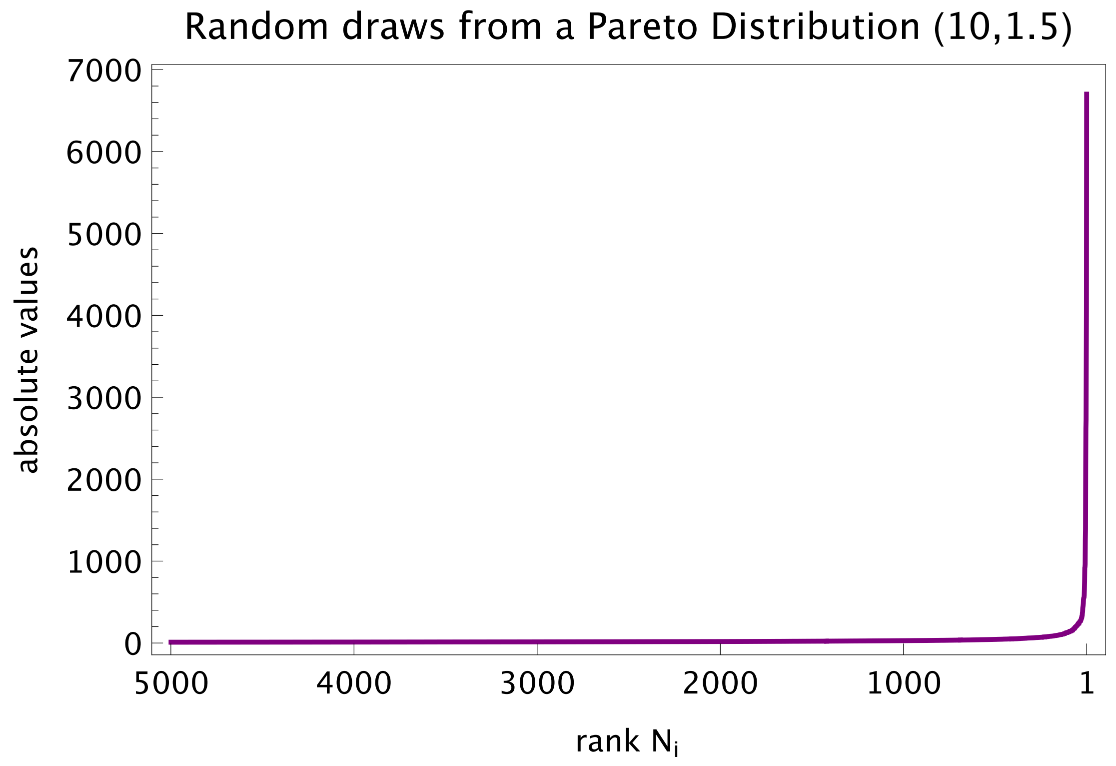

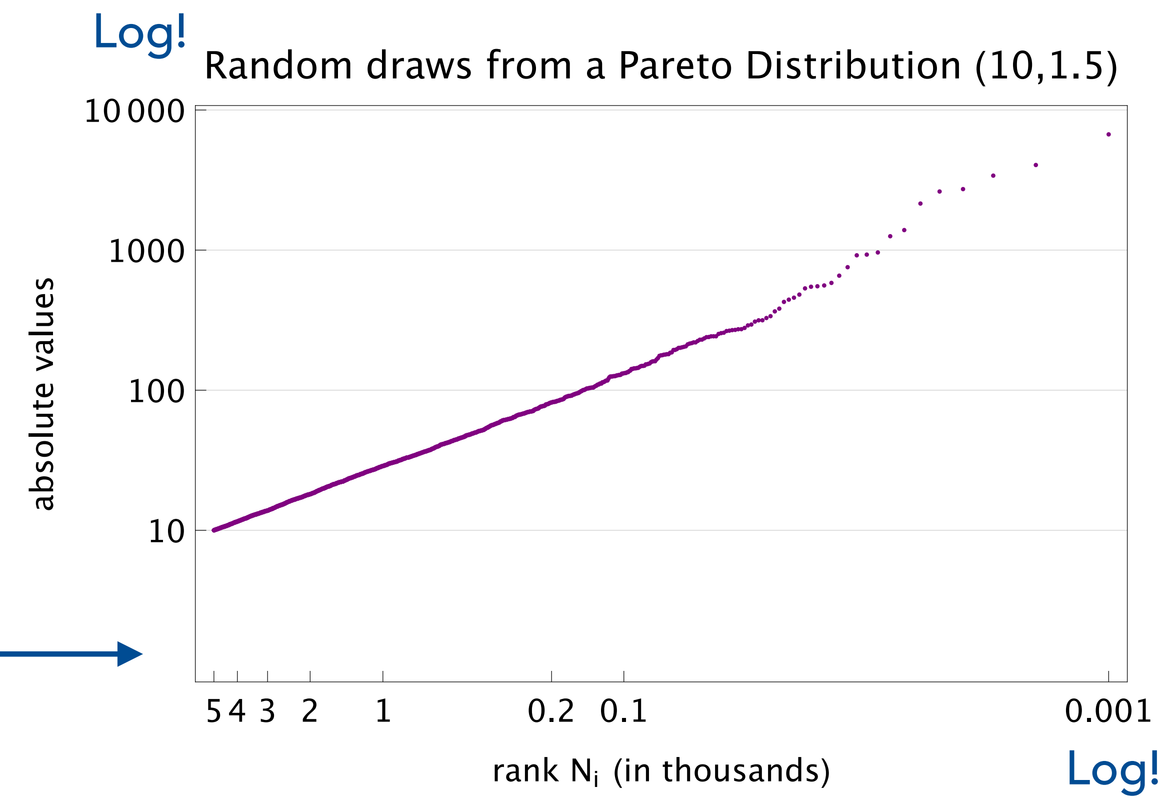

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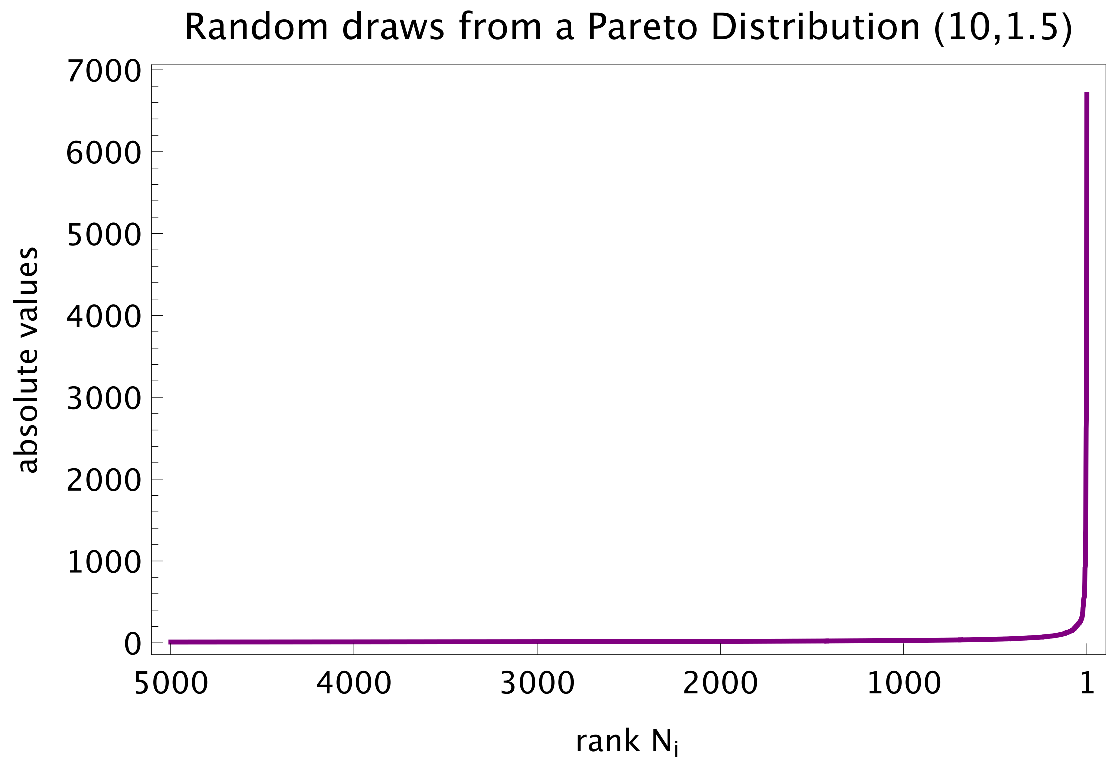

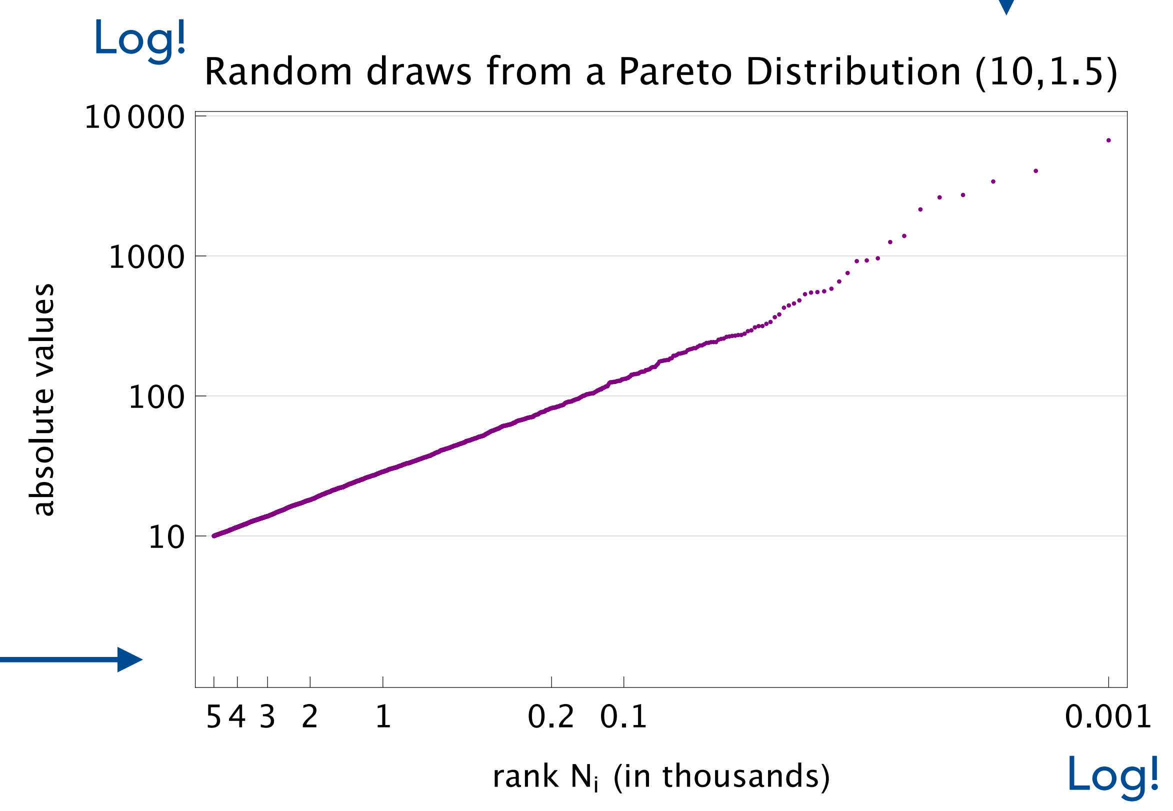

Talking about power laws

Talking about power law distributions?

- Three main problems when talking to lay audiences / students / media about power law distributions
  - Large number problem
  - The „what is a distribution?"-problem
  - The „fancy properties of power-law distribution"-problem

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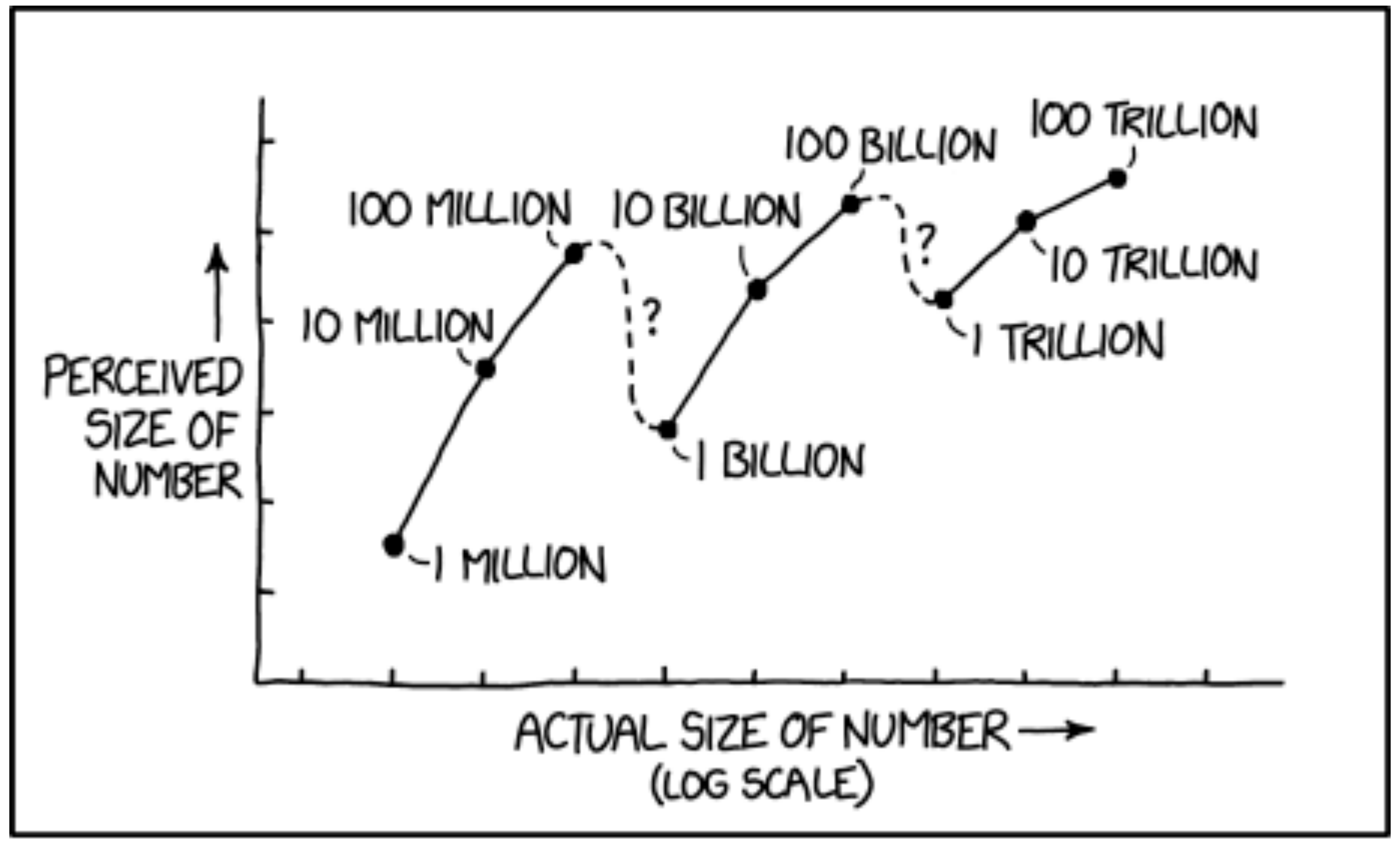

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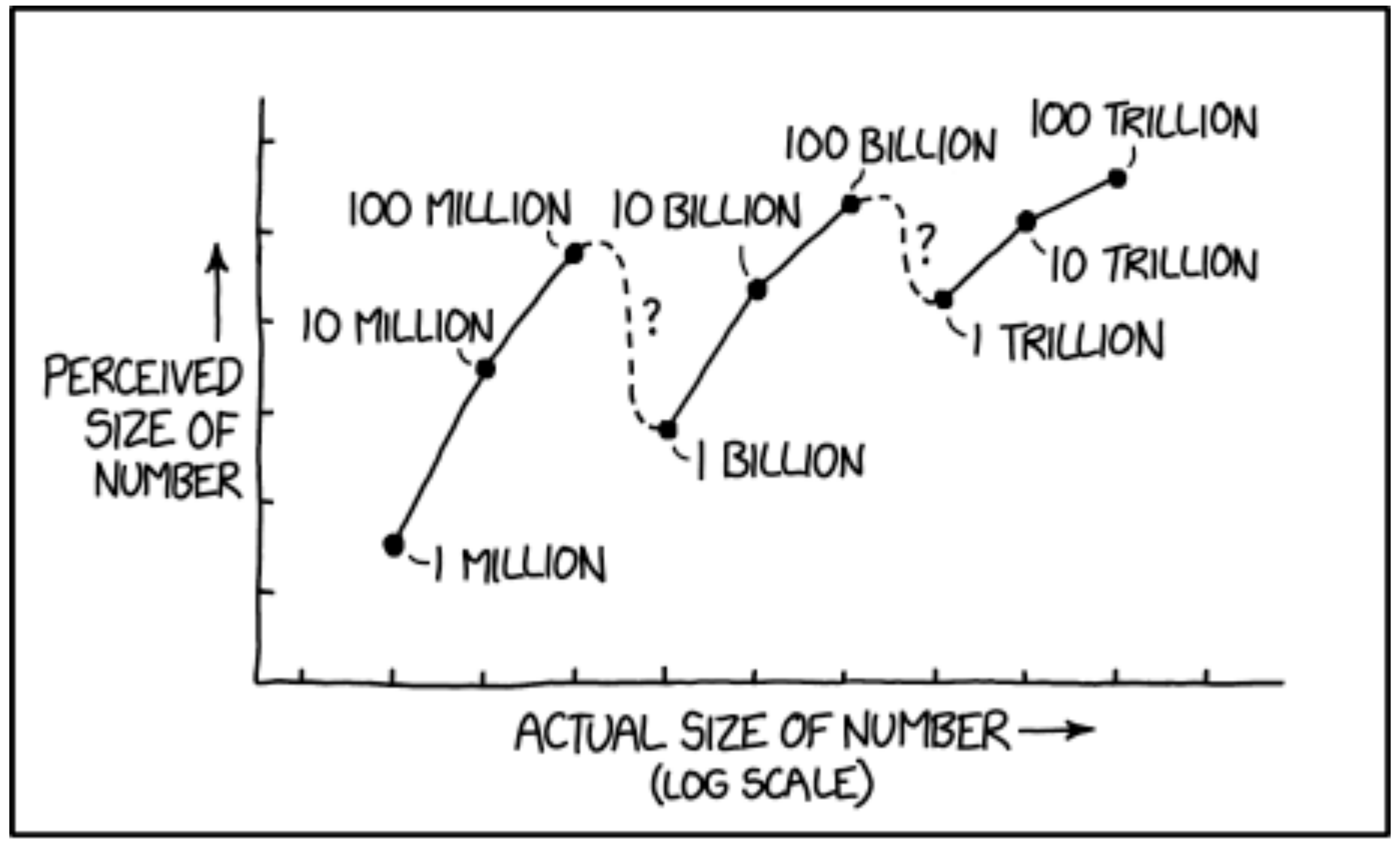

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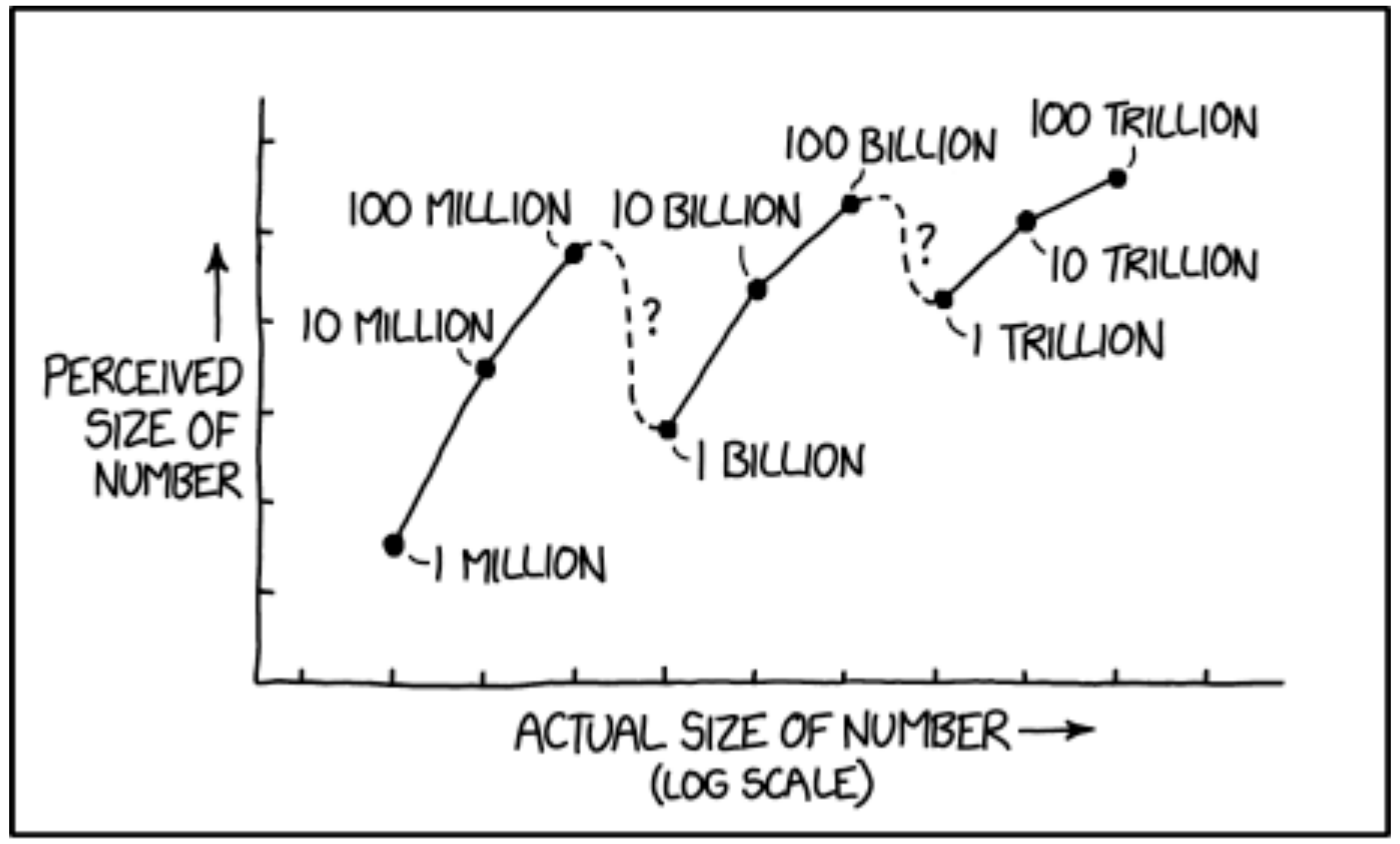

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- „Give examples for the mysterious"
  - What would *„I met a huge guy"* or *„I saw a huge tree"* mean, if these variables were power law distributed? Give numerics – compare mean to max!

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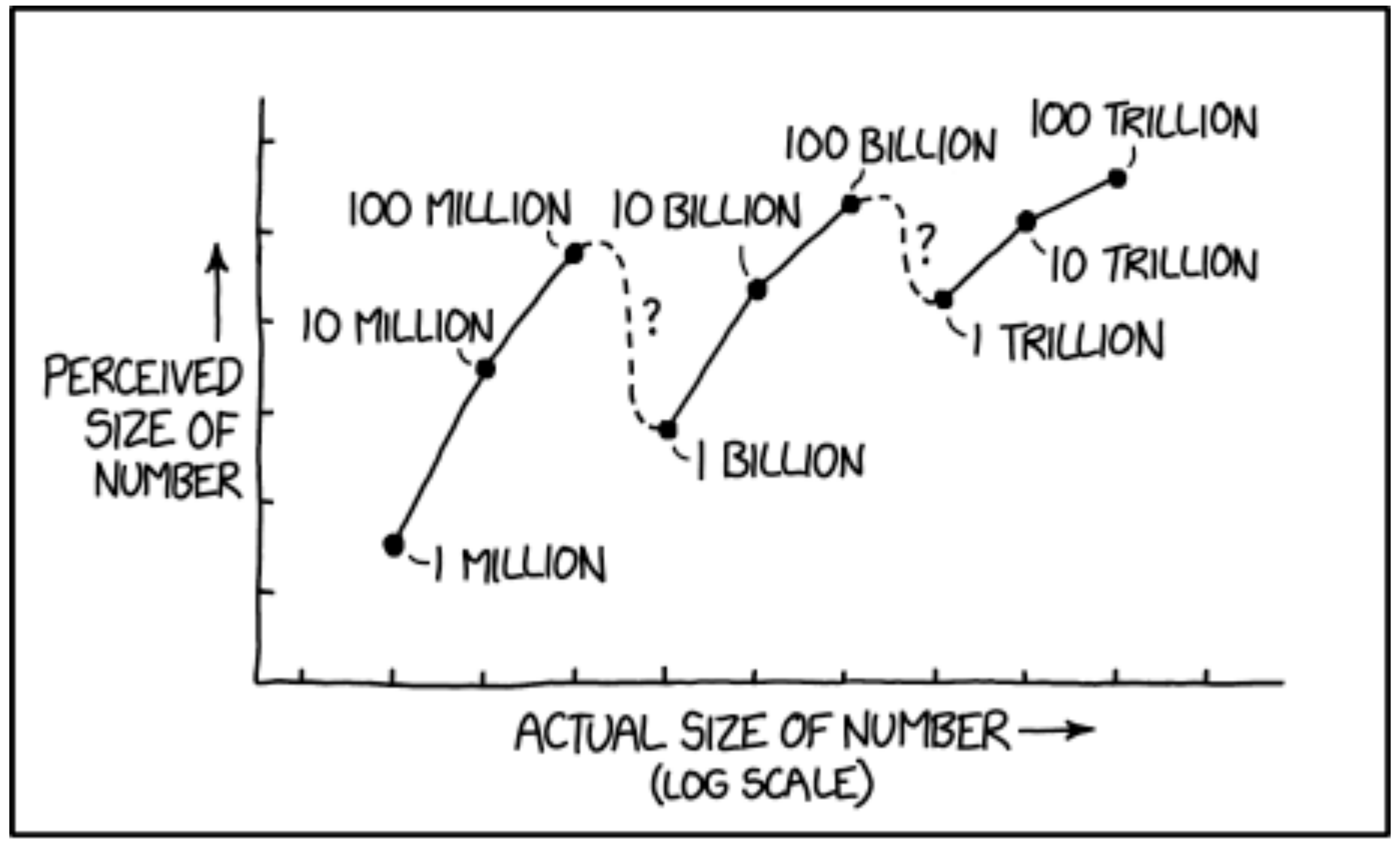

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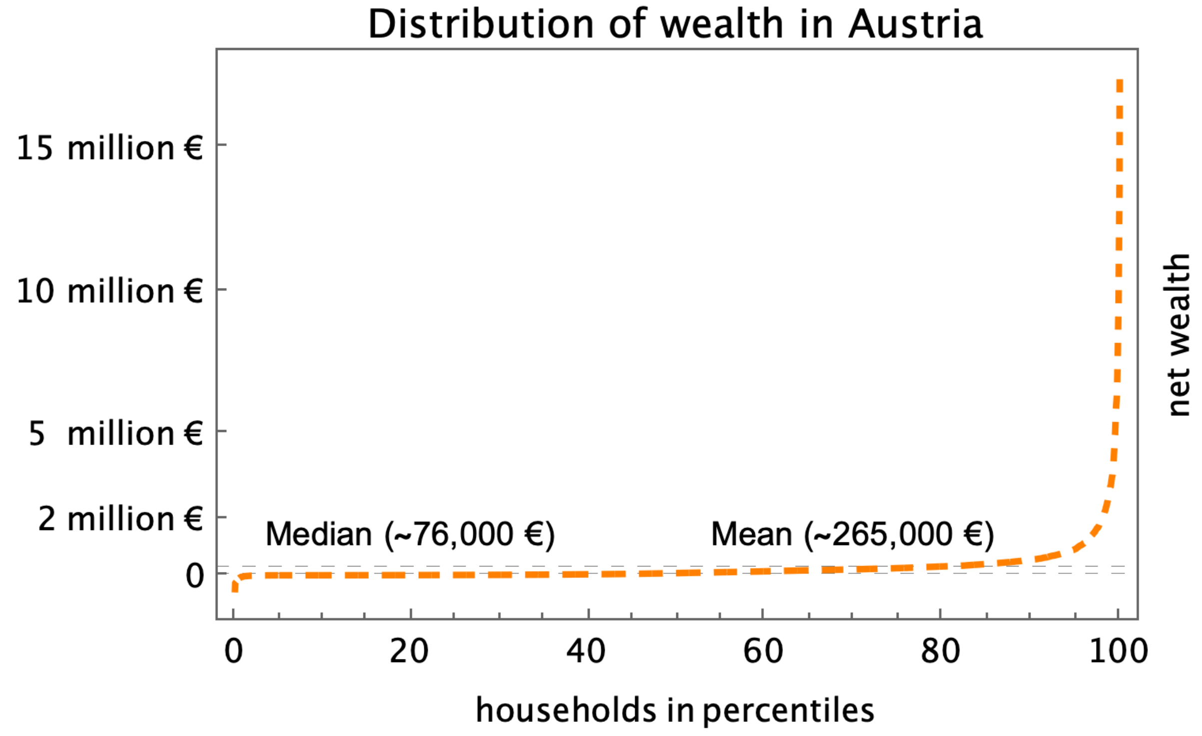

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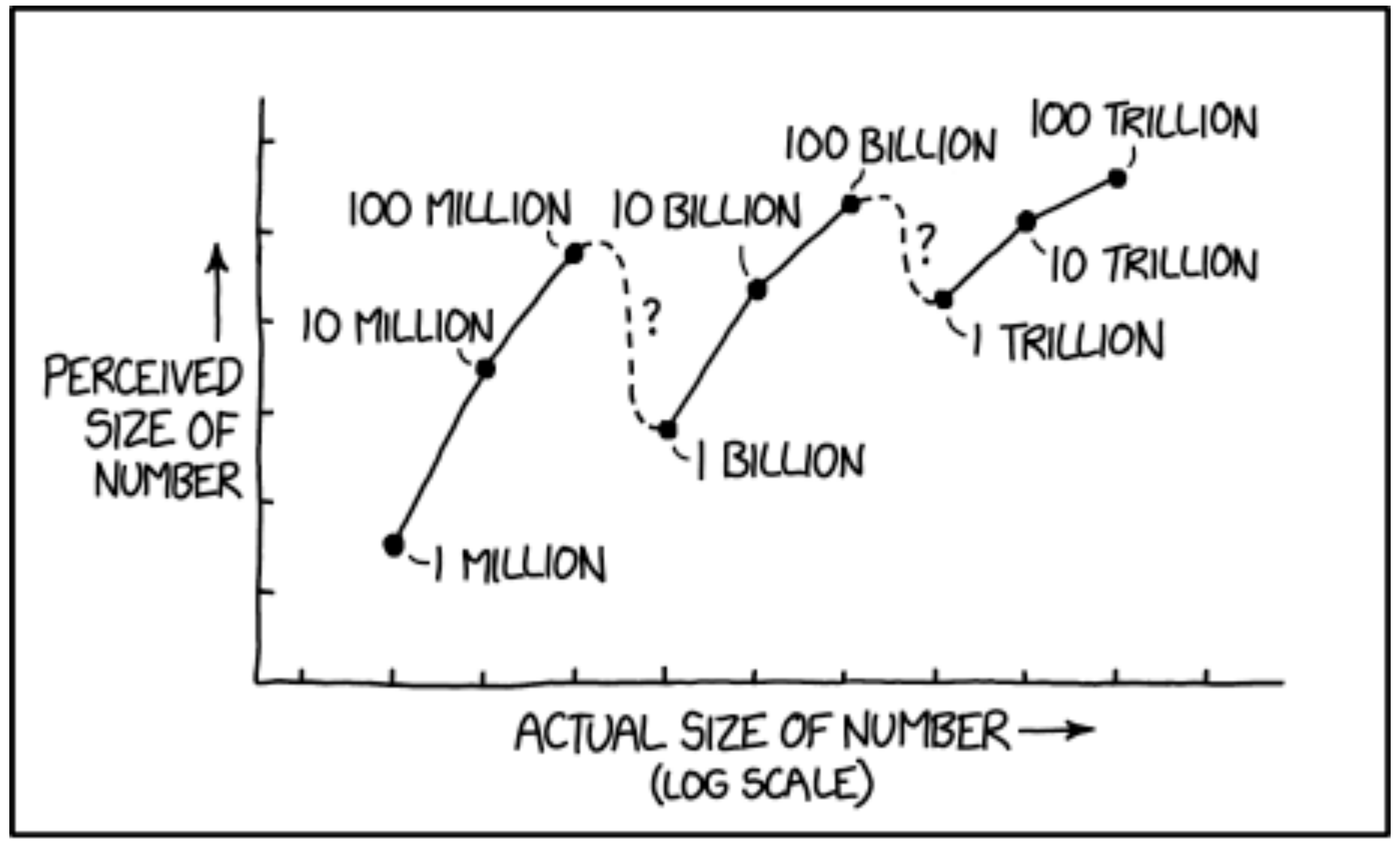

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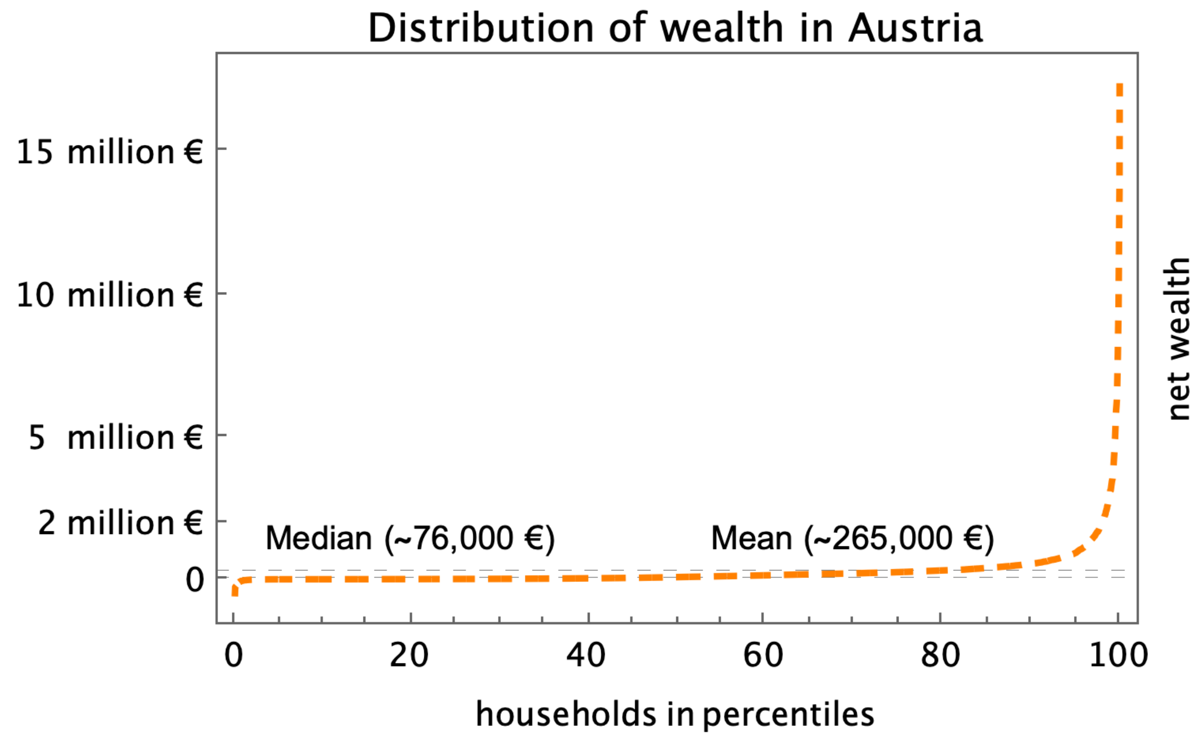

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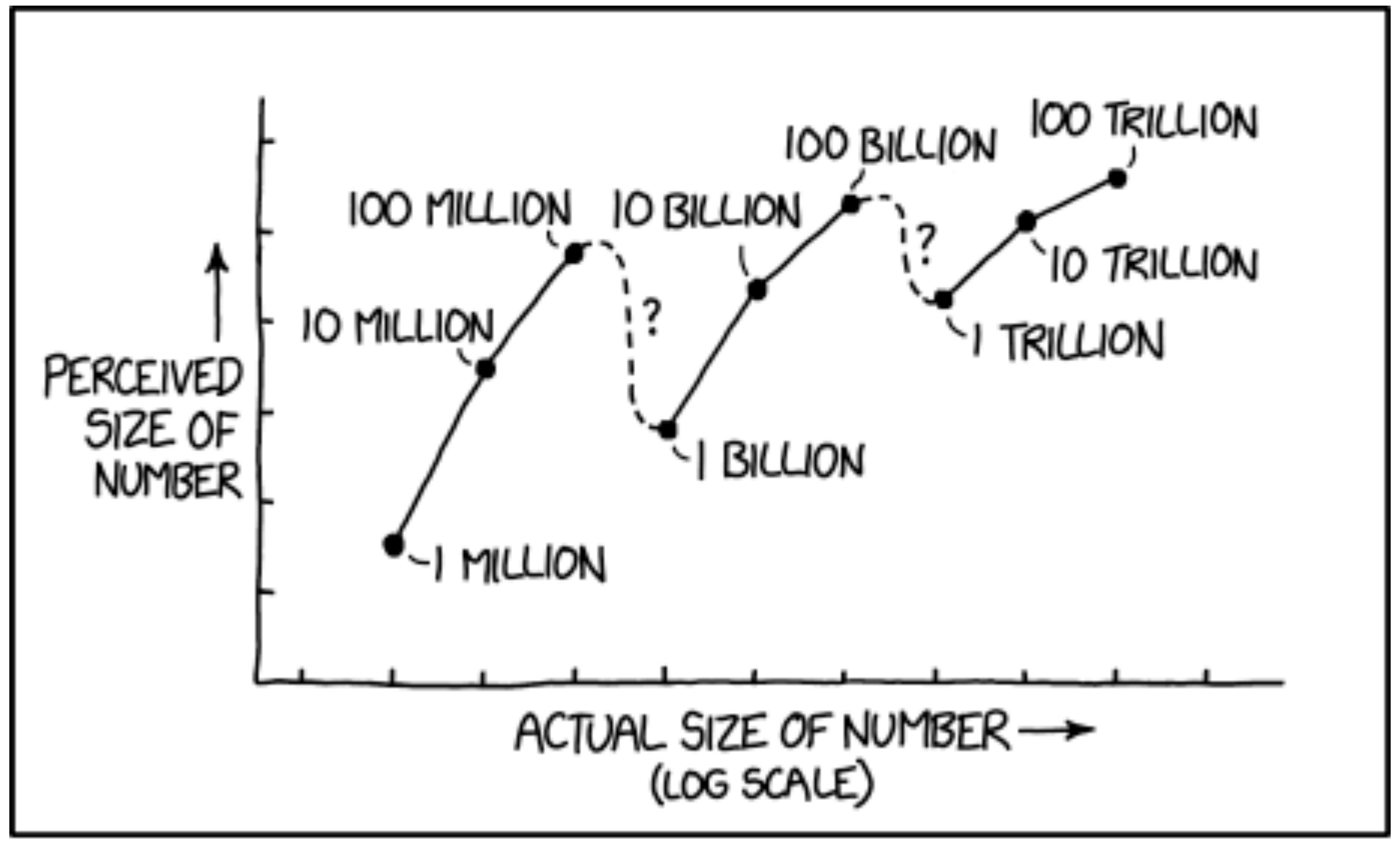

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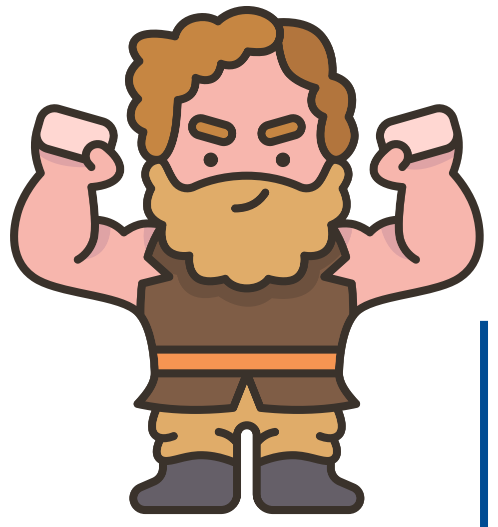

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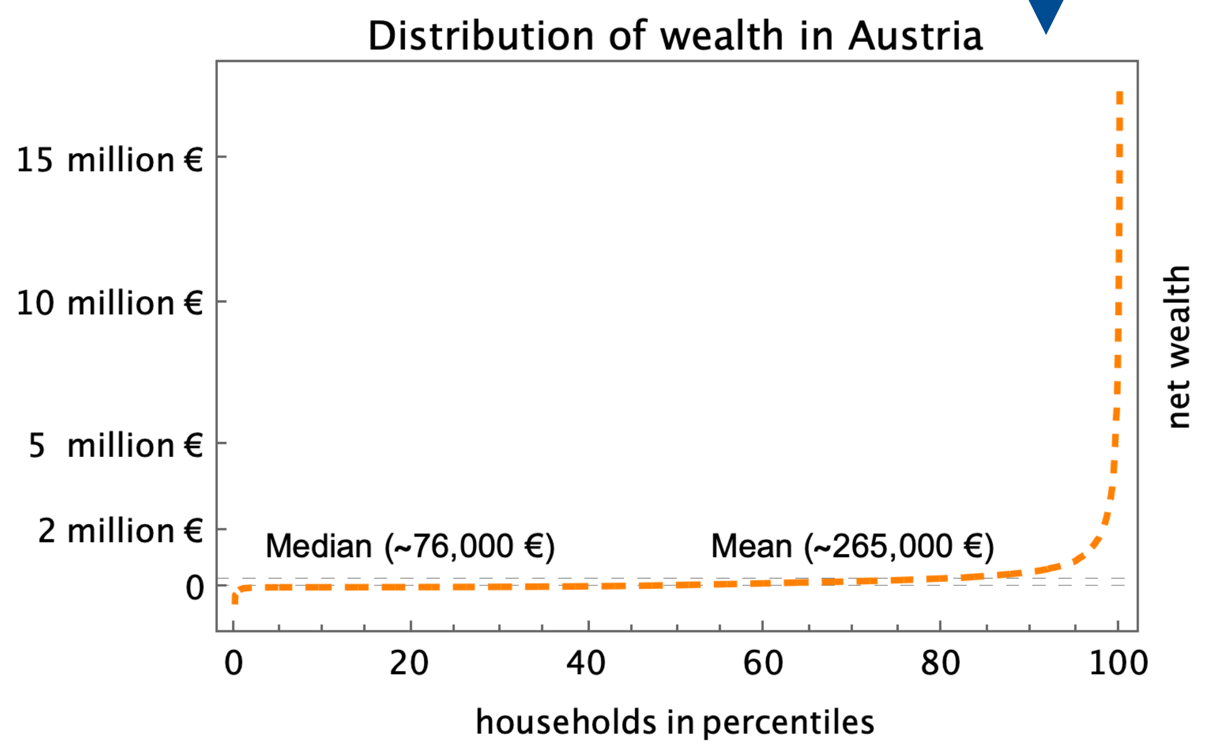

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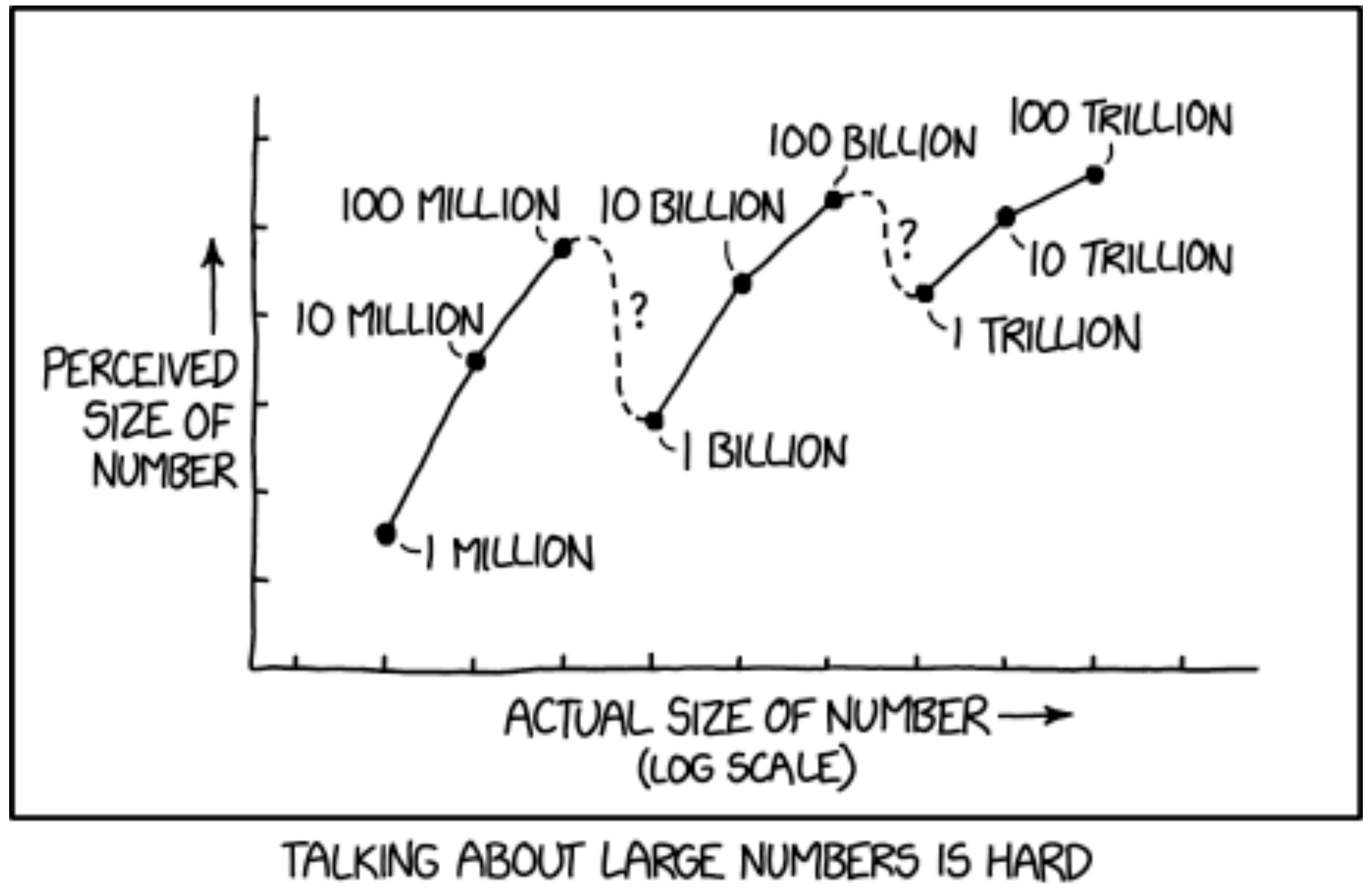

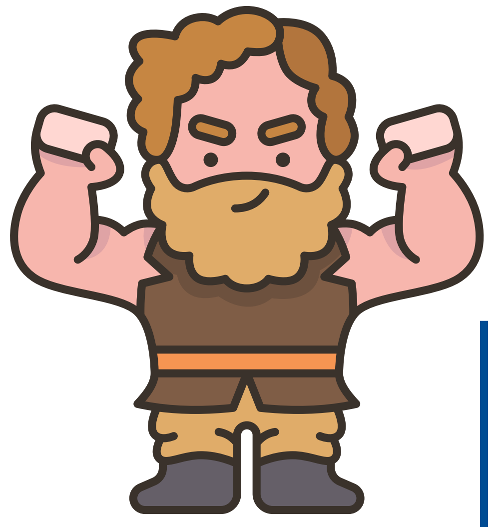

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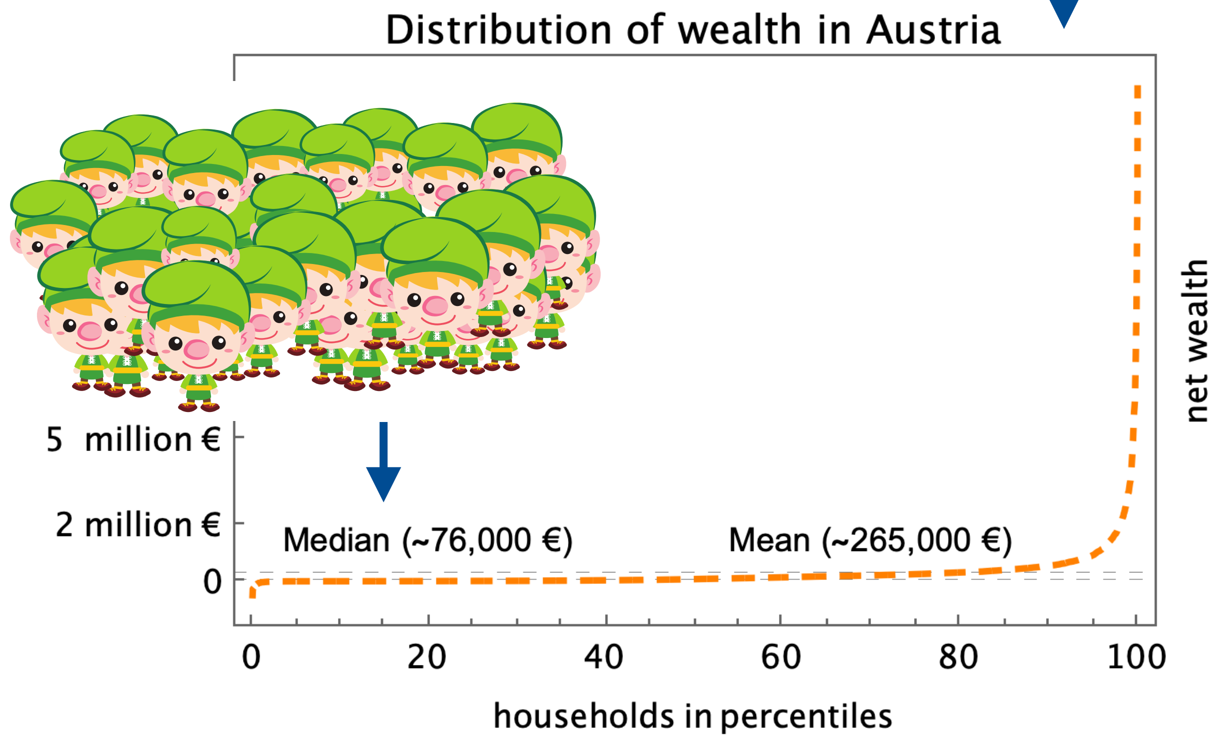

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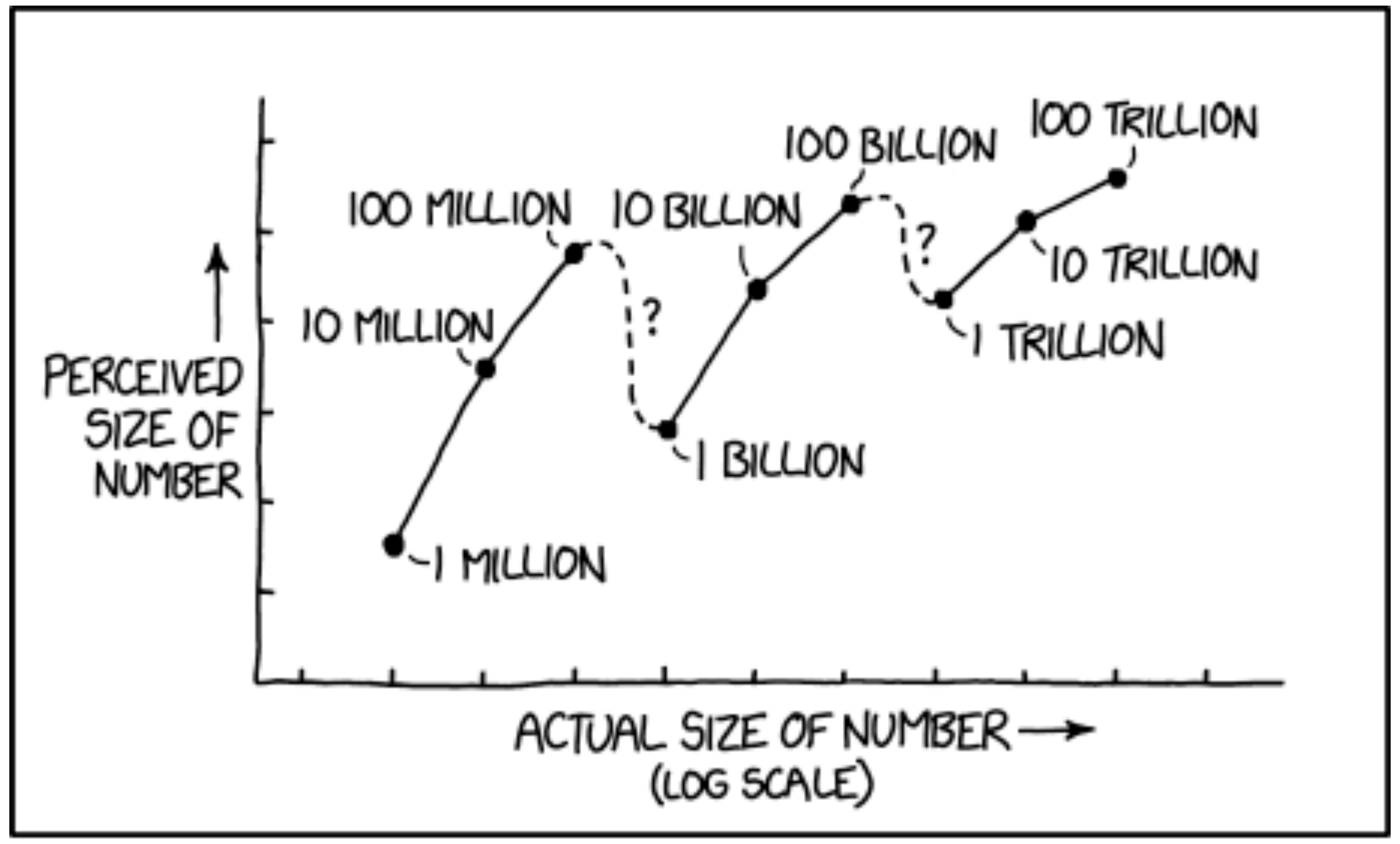

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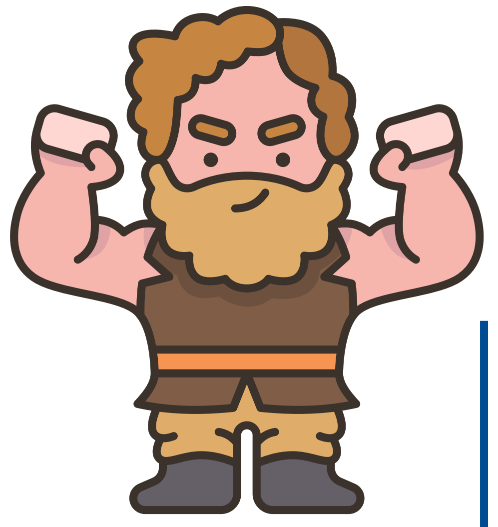

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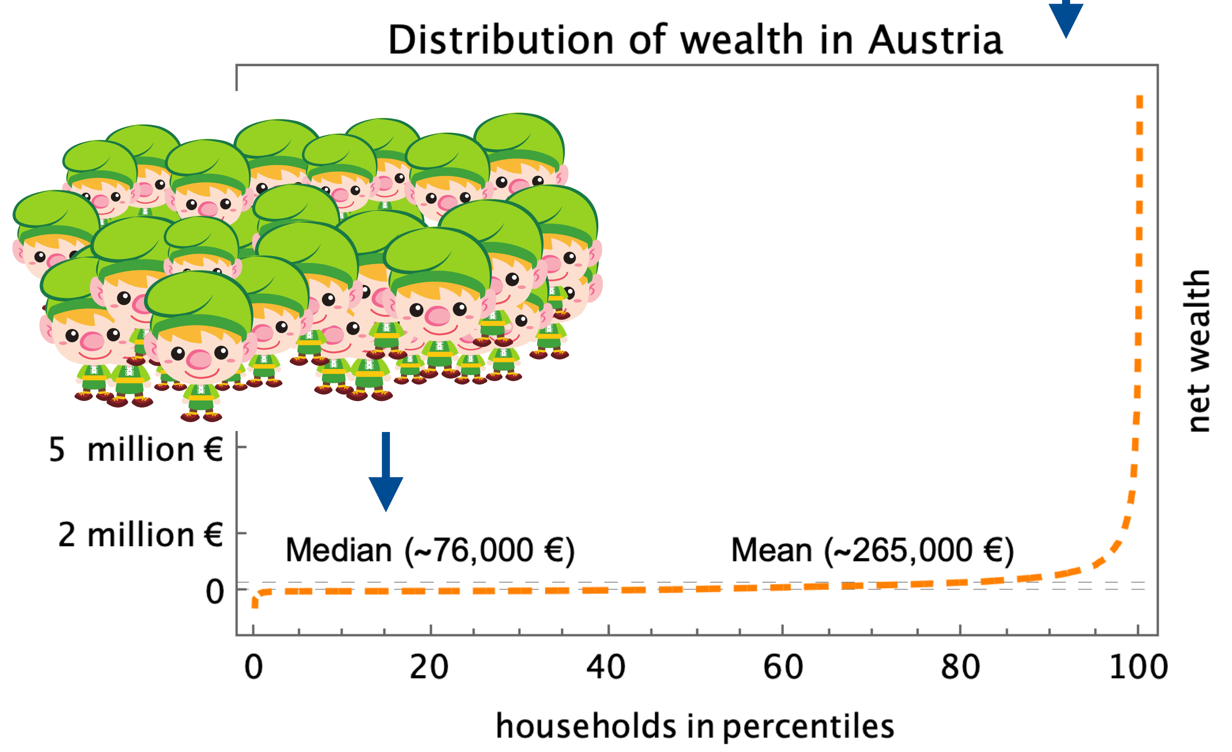

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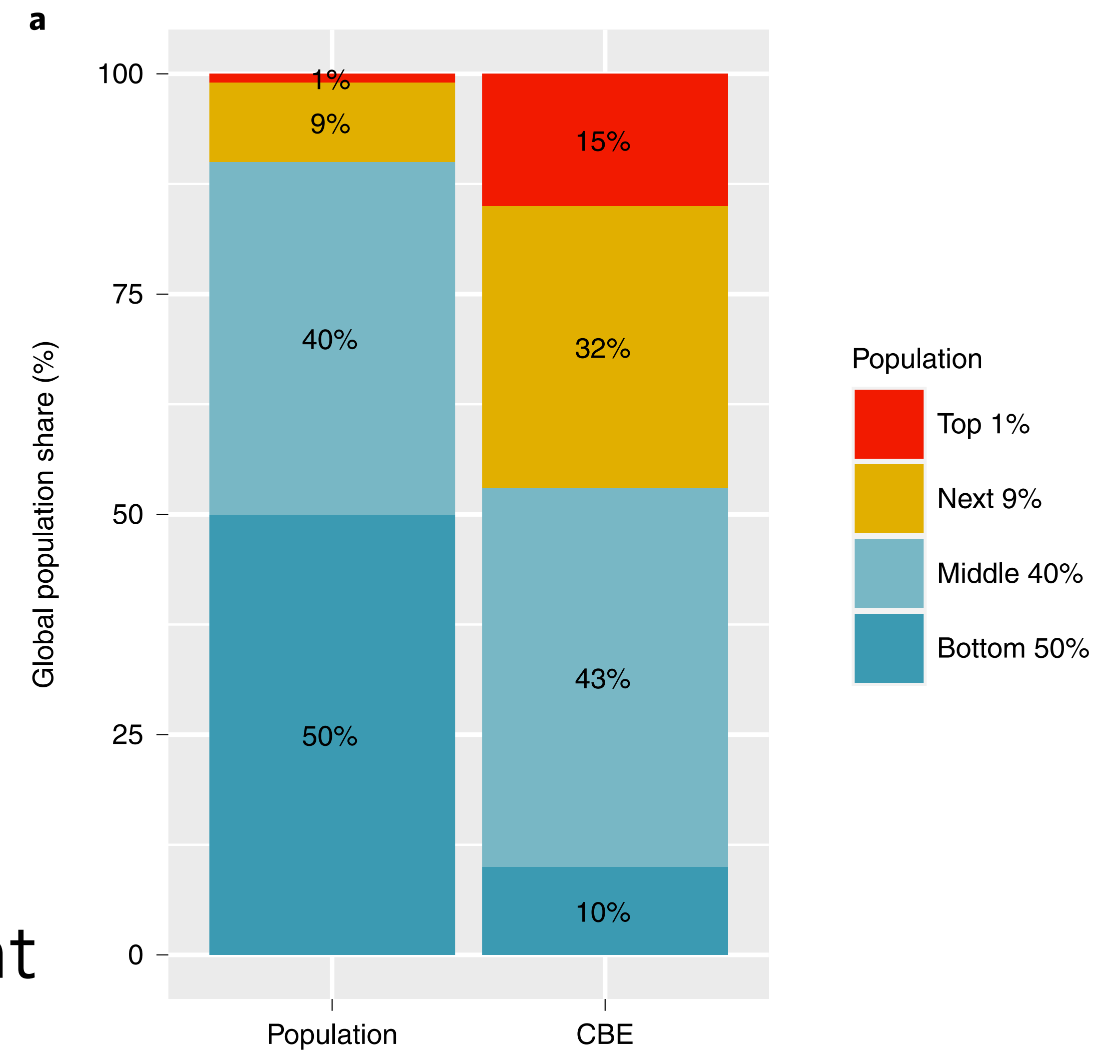

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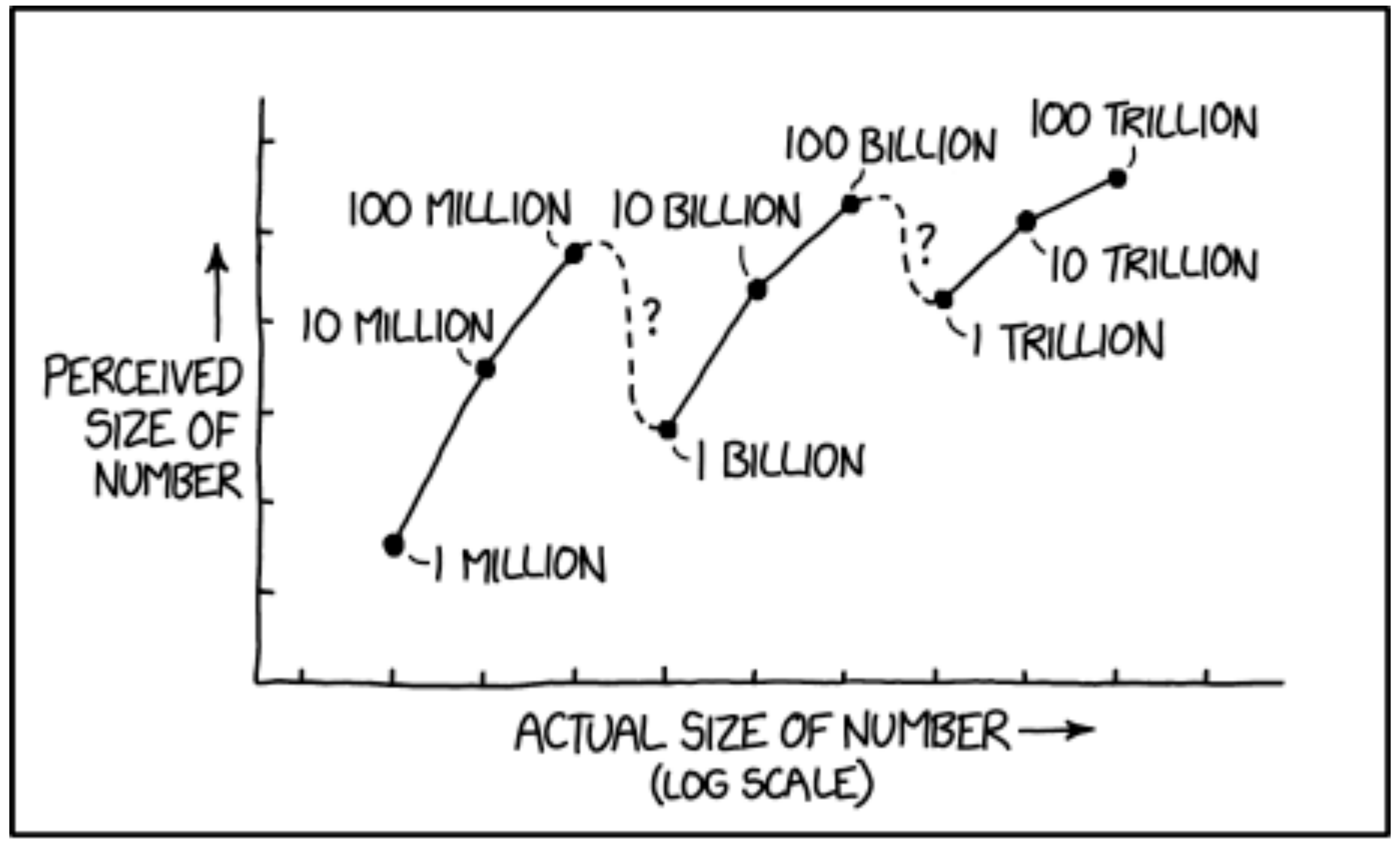

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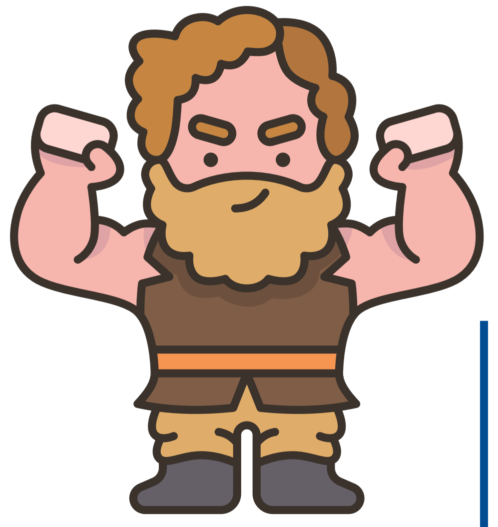

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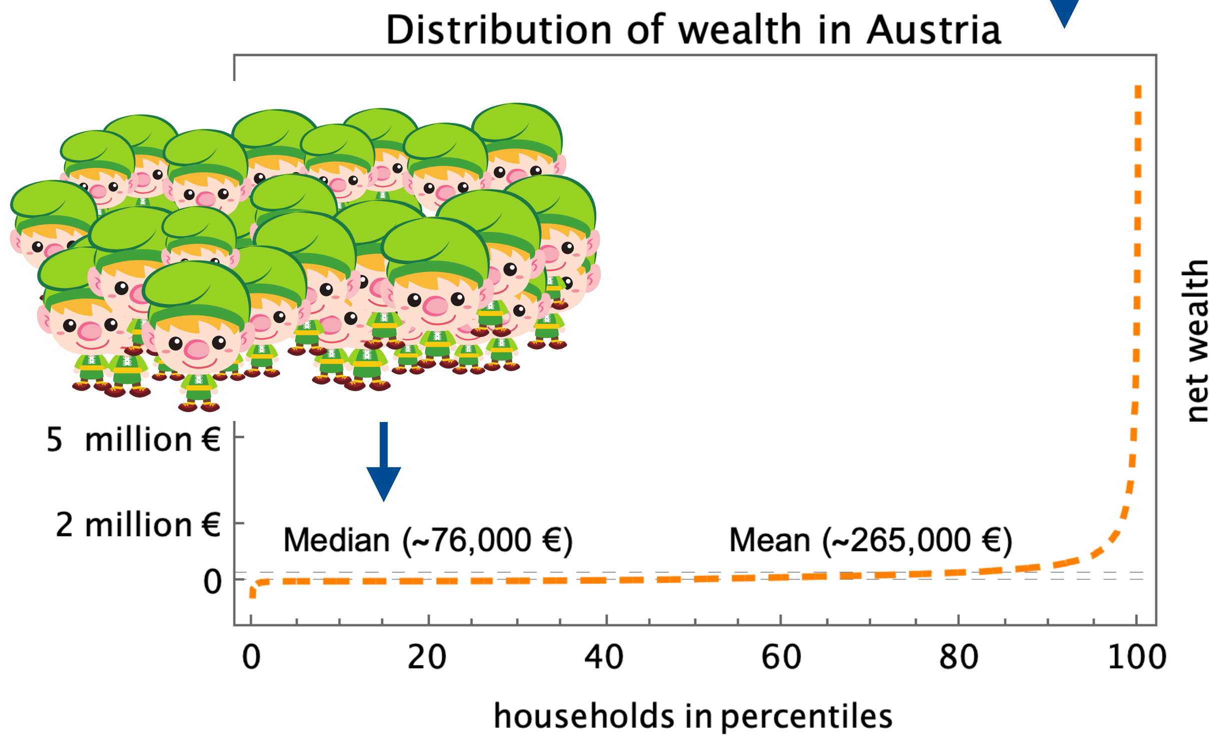

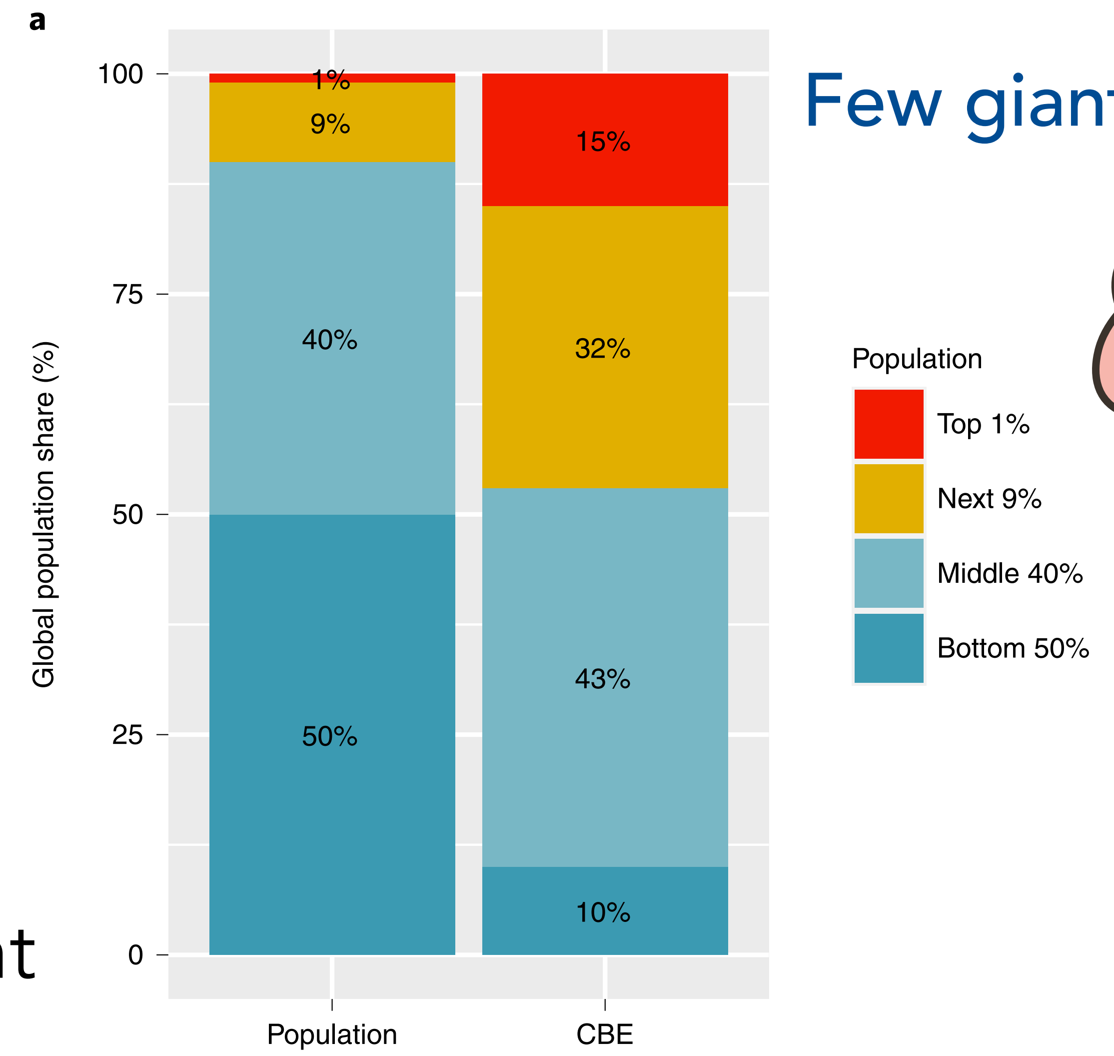

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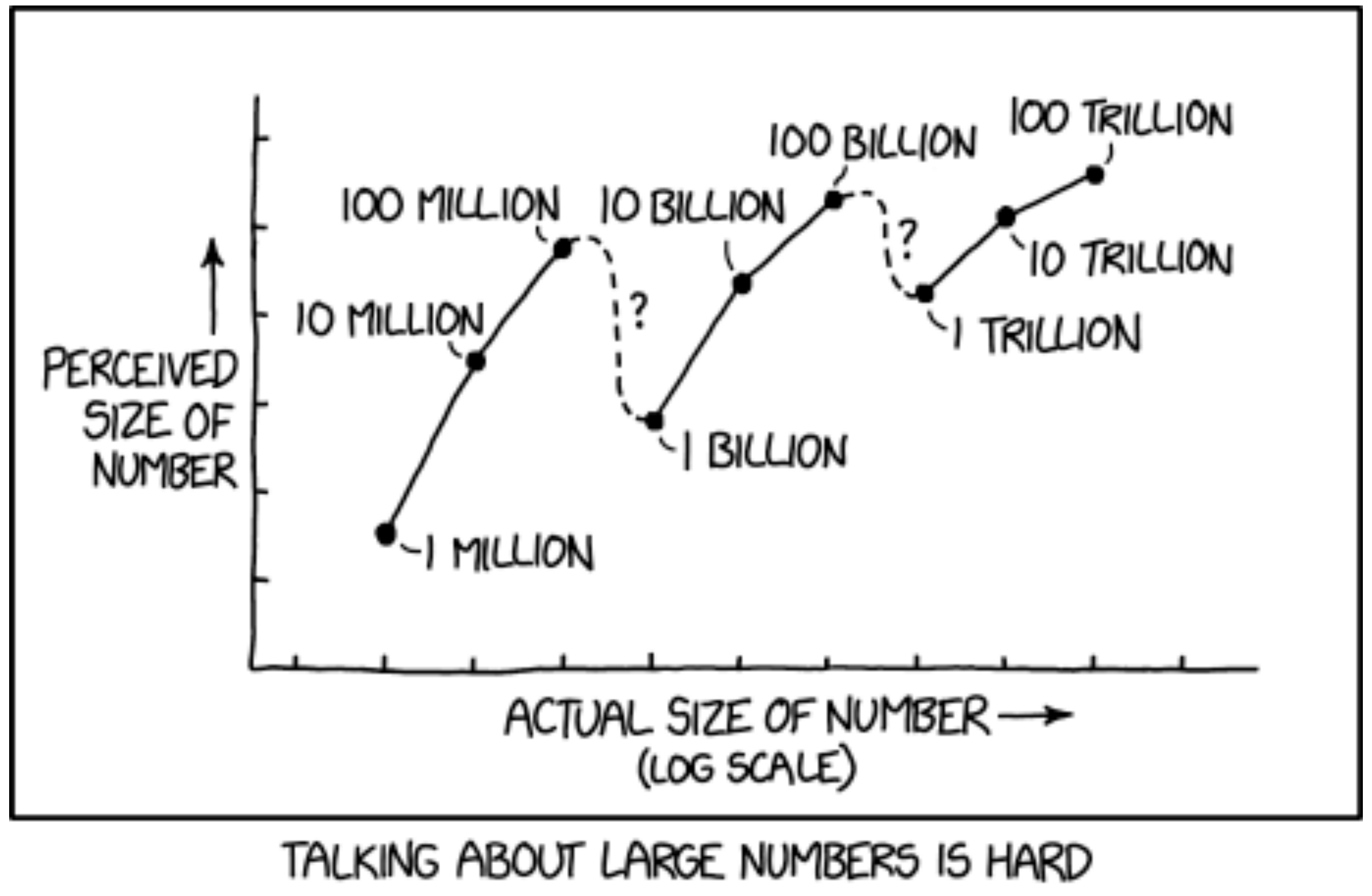

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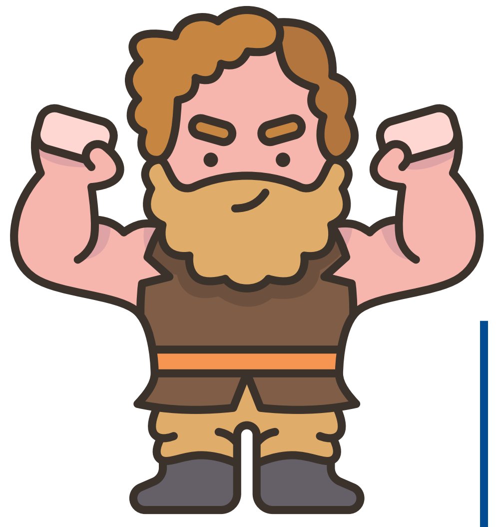

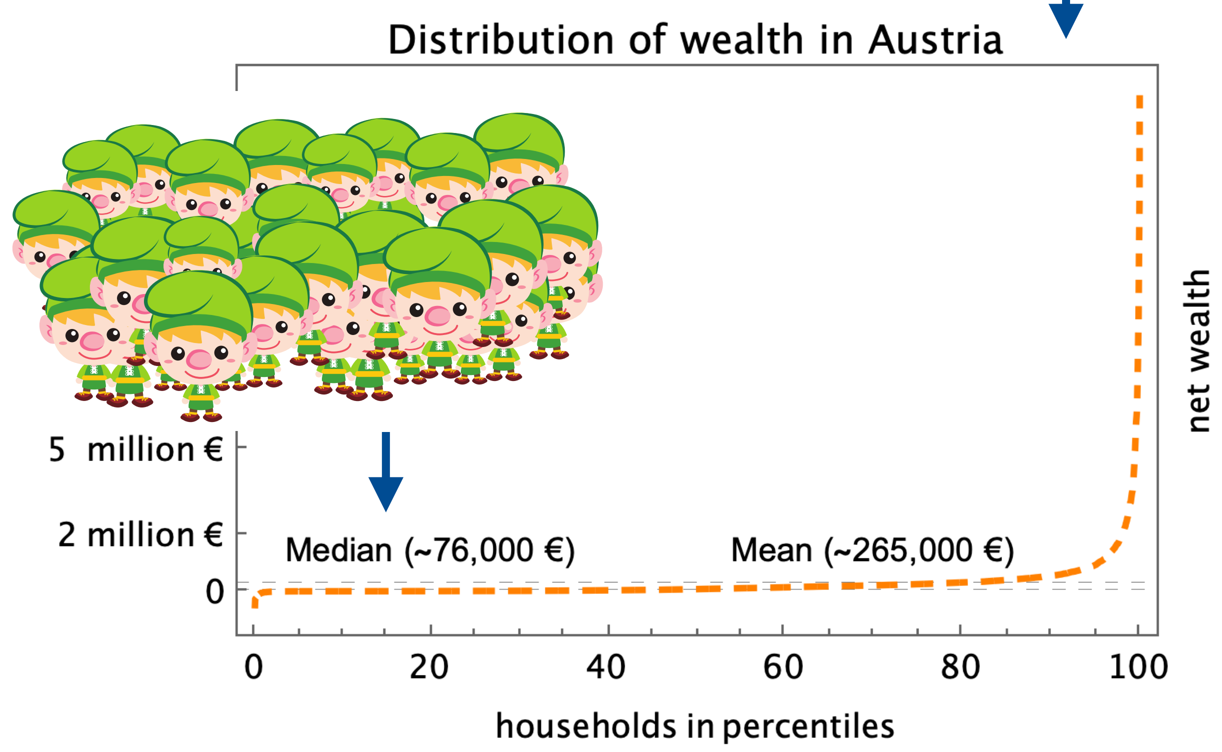

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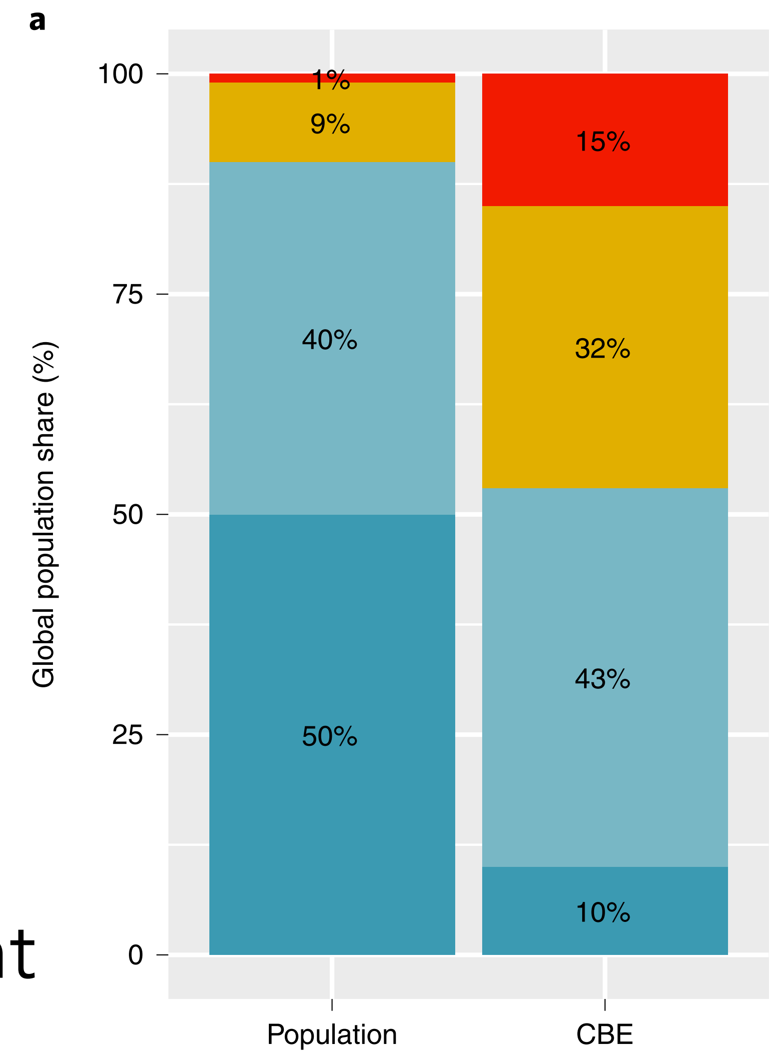

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Generative mechanisms

How do power law distributions emerge?

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  - But what happens in terms of actual processes, that drives this (regular) pattern?

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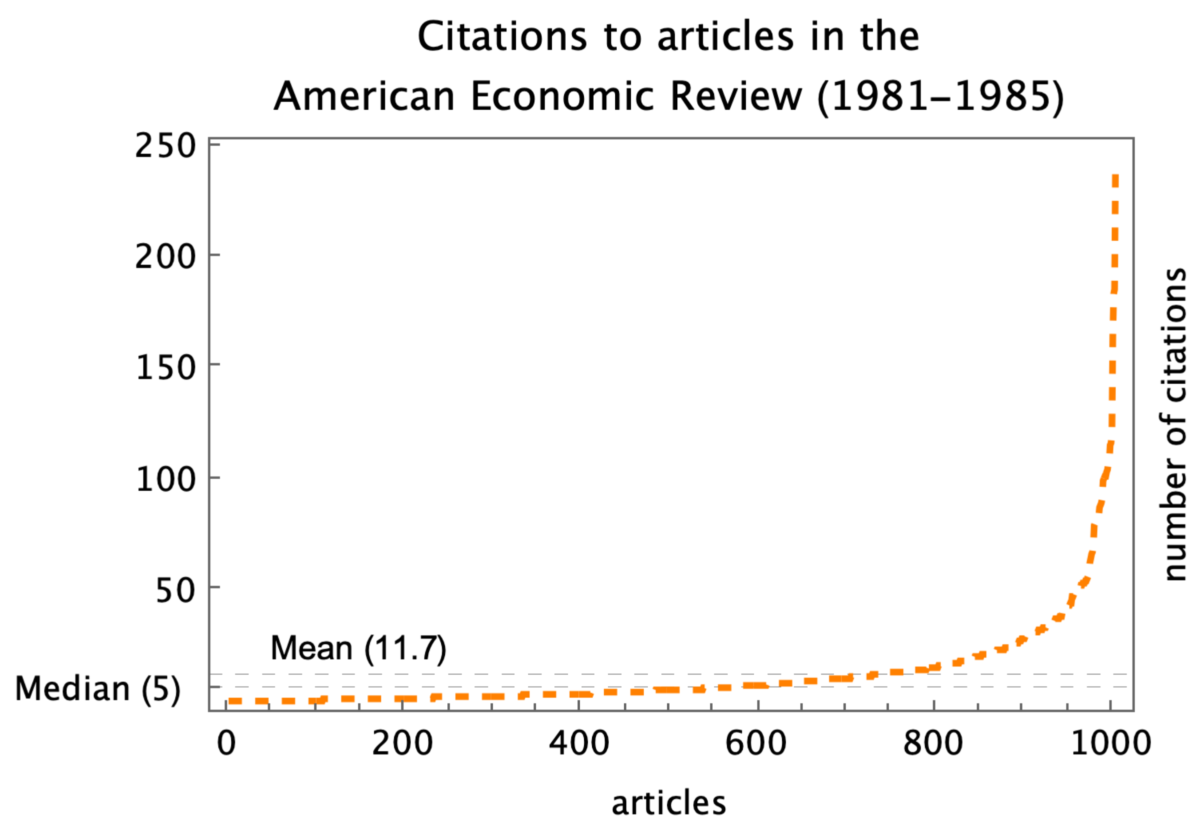

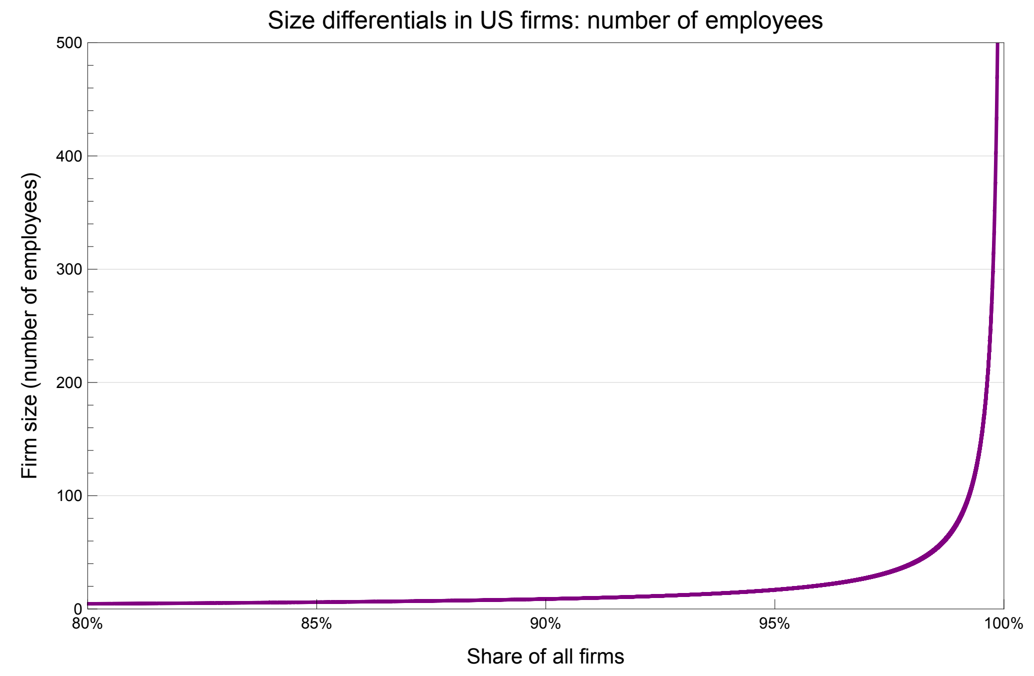

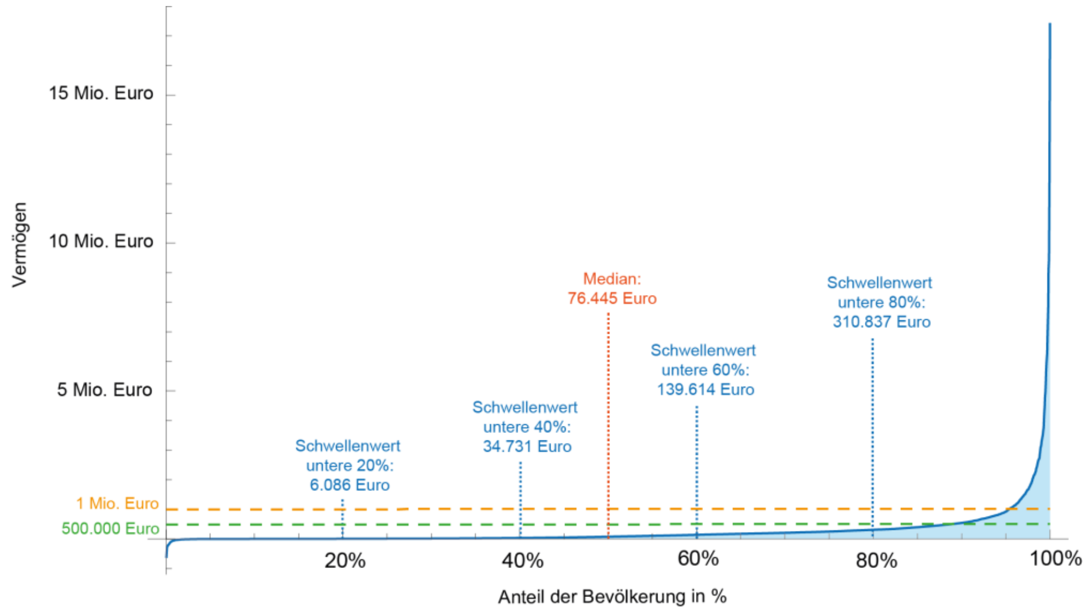

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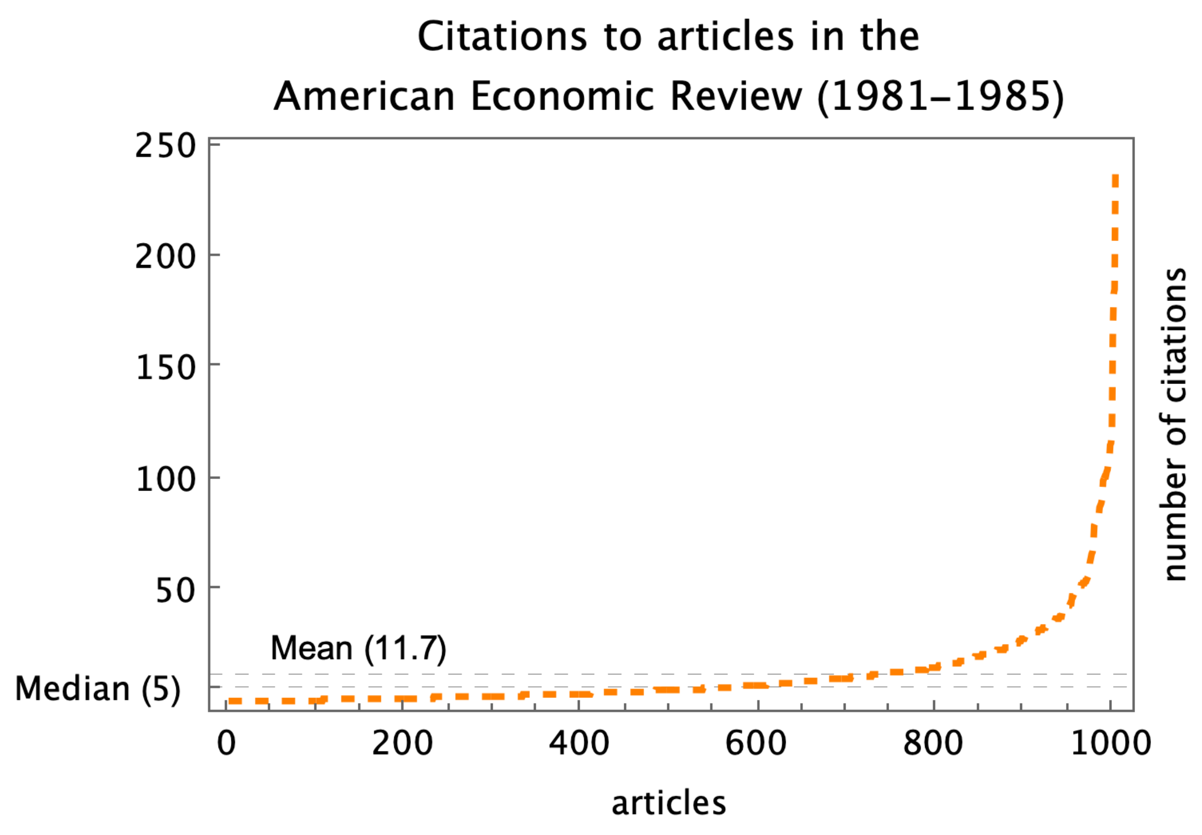

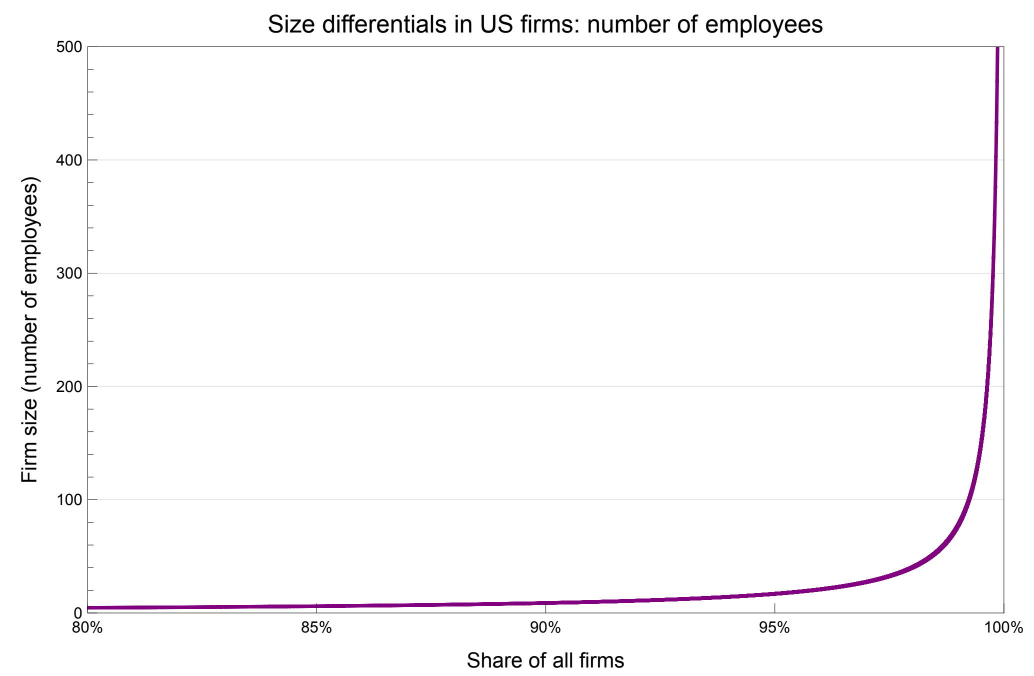

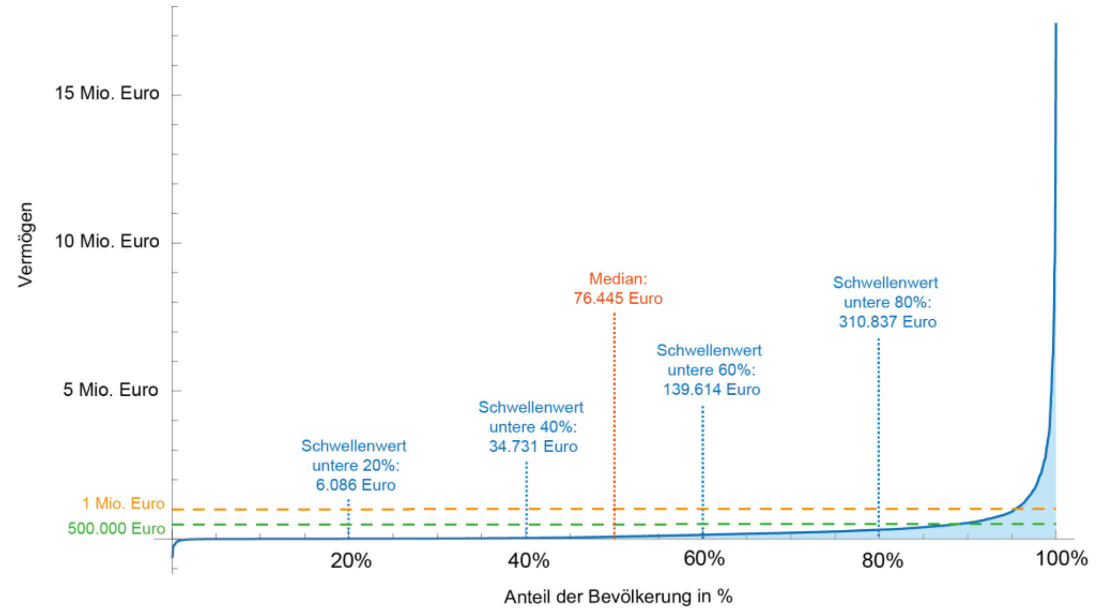

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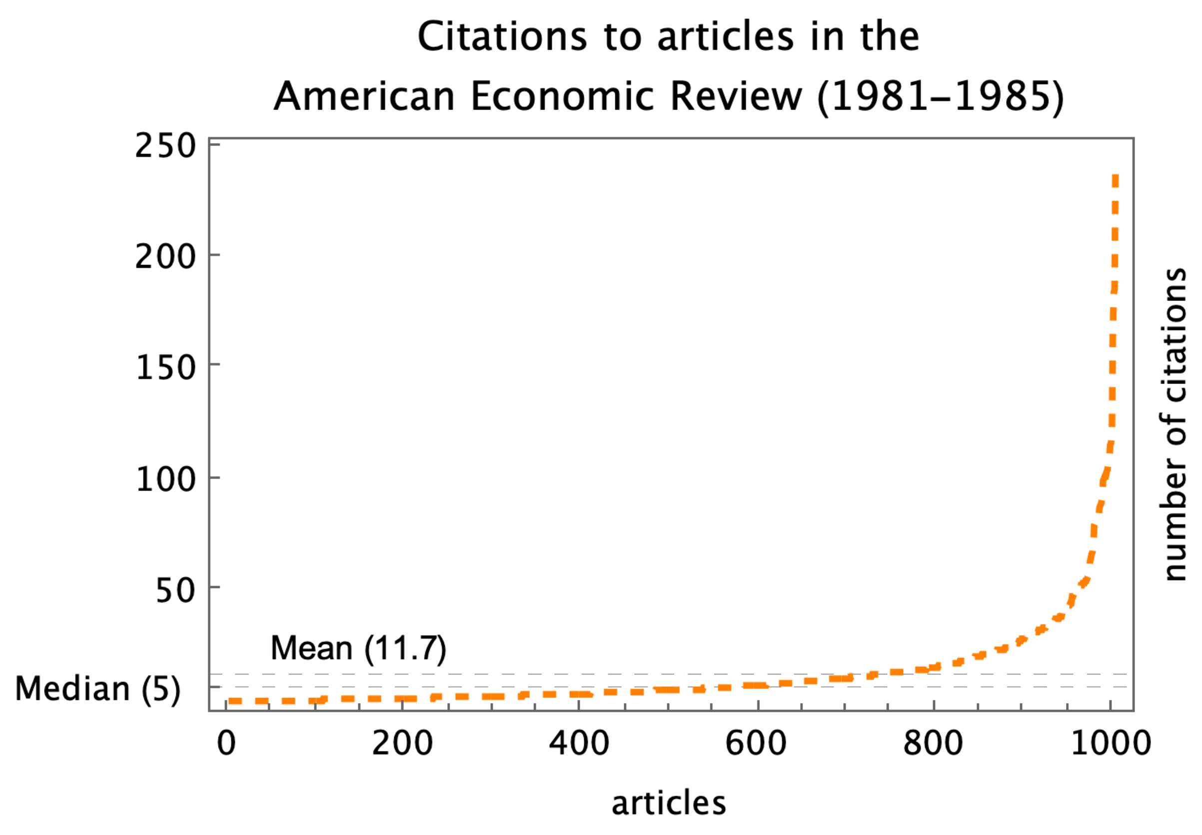
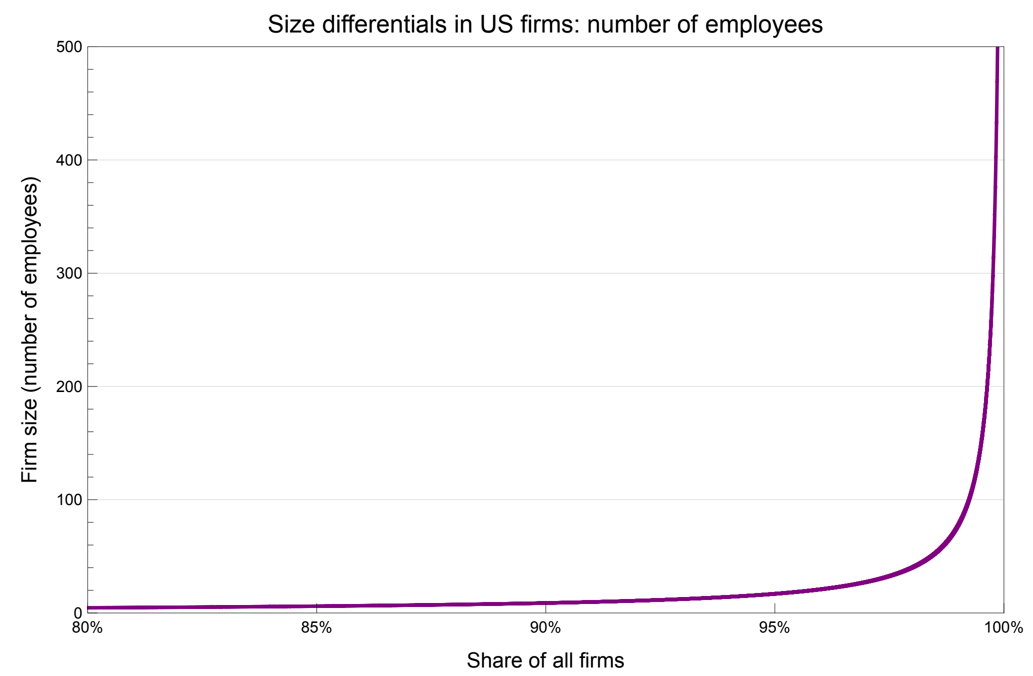
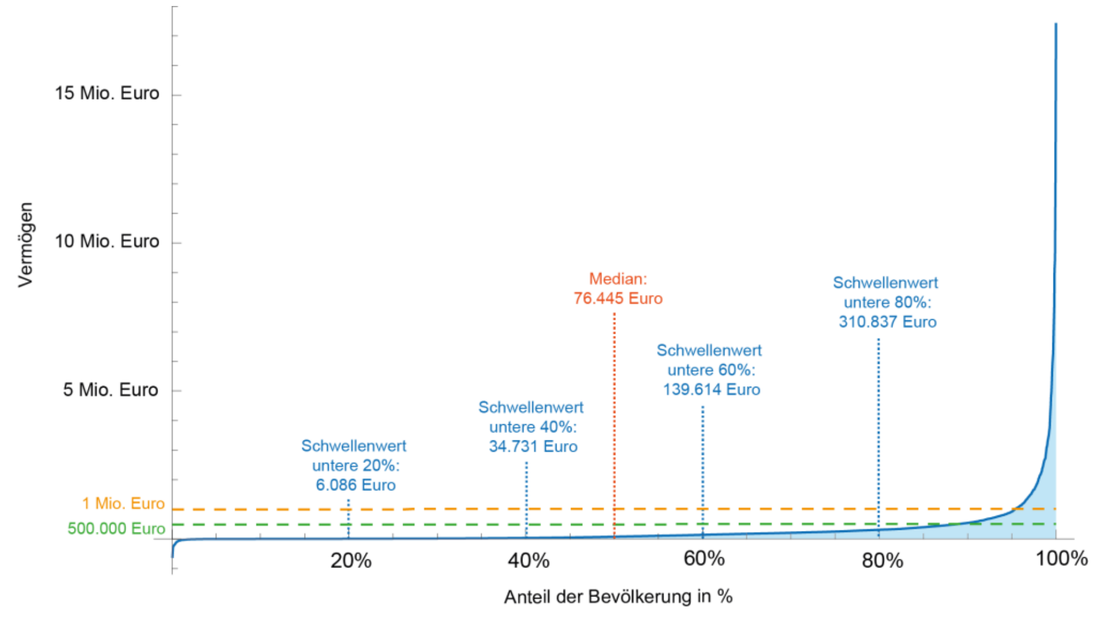
$$\mathbb{P}(wealth \geq w_i) = w_i^{-\frac{\delta}{\lambda}} \leftarrow x_i = \frac{log(w_i)}{\lambda}$$

These two can be found in finance!

Inequality is stable & steady-state exists!







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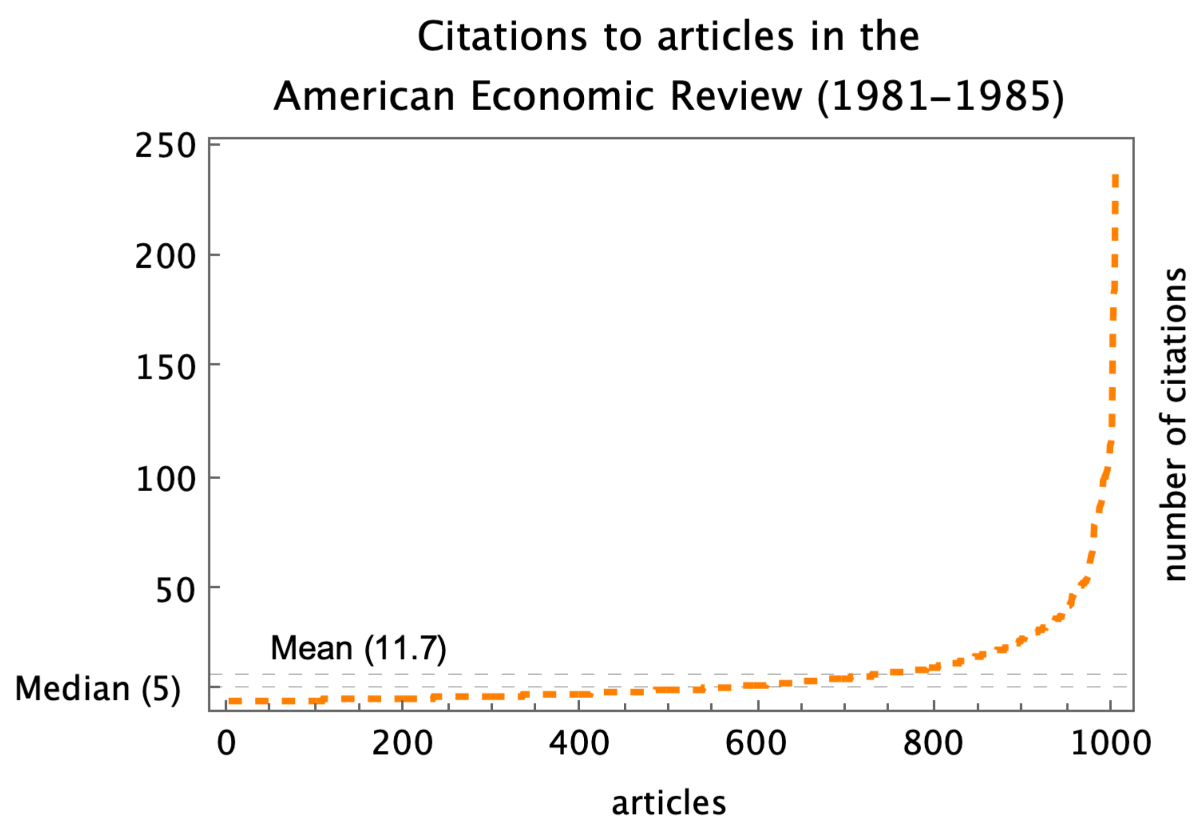

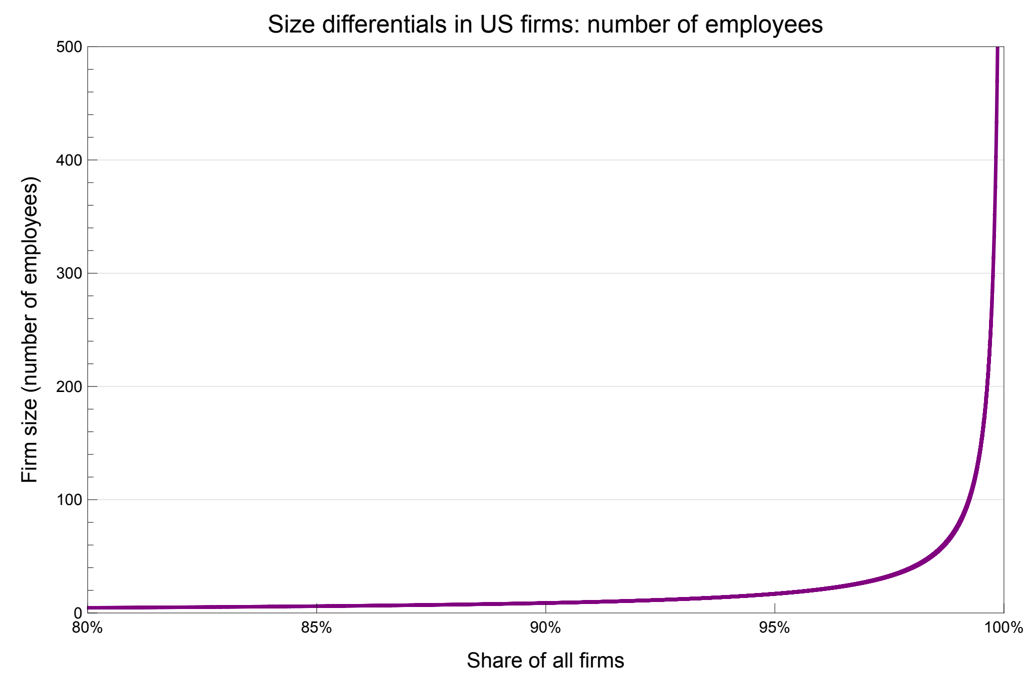

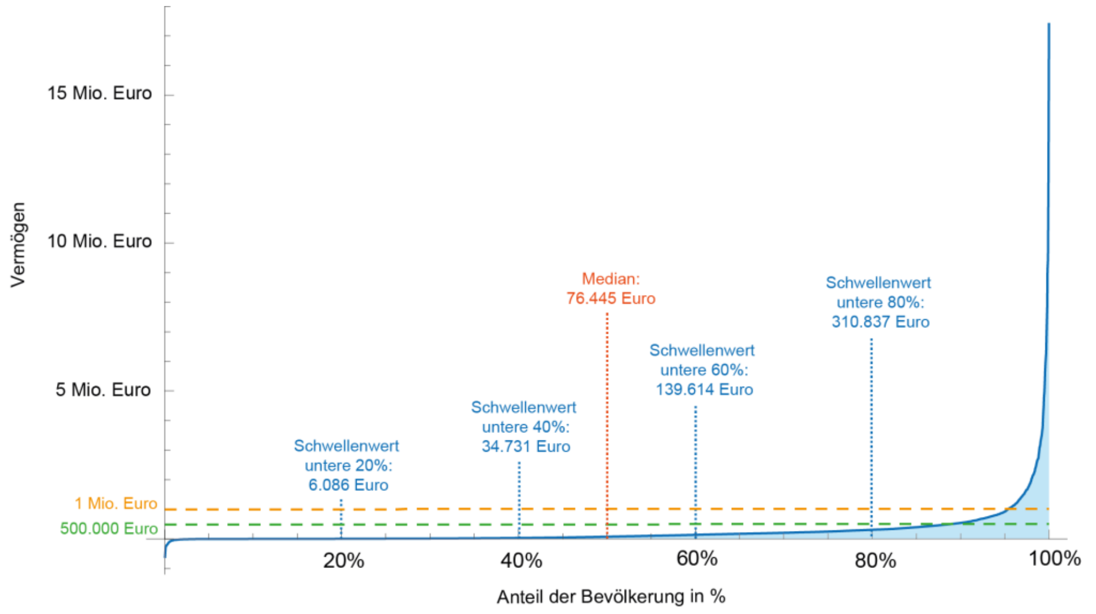

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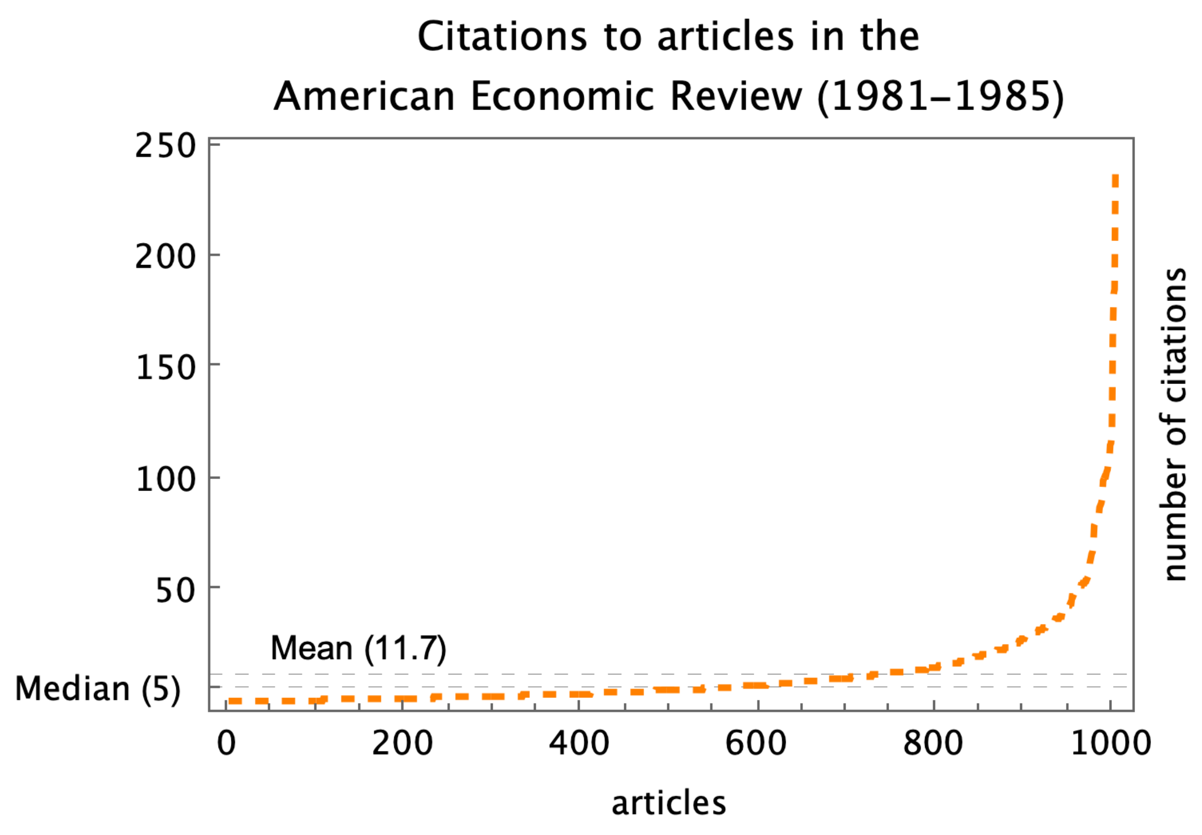

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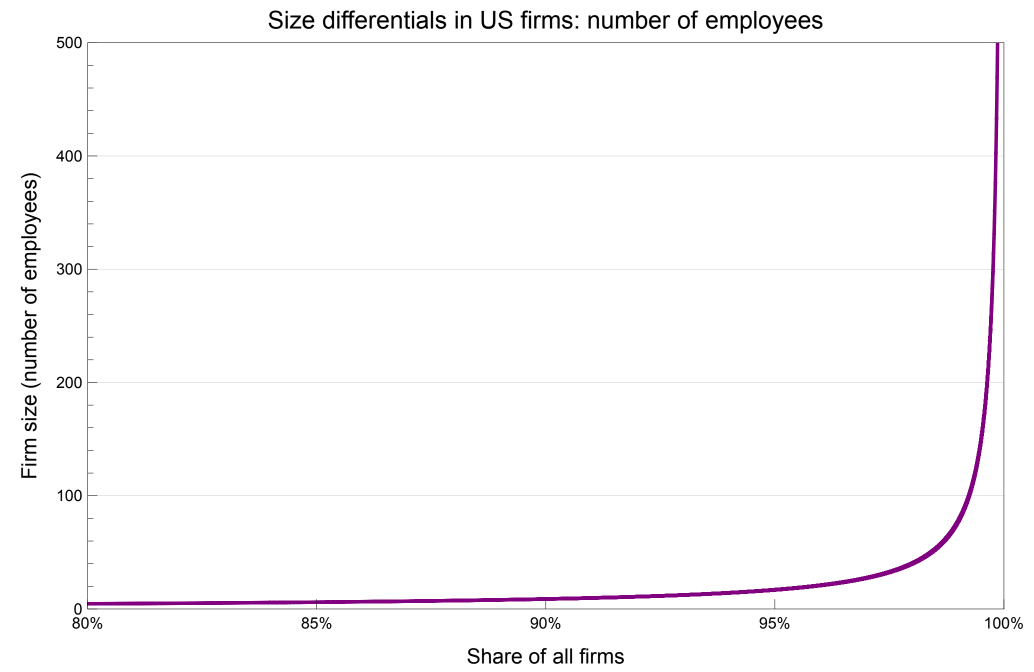

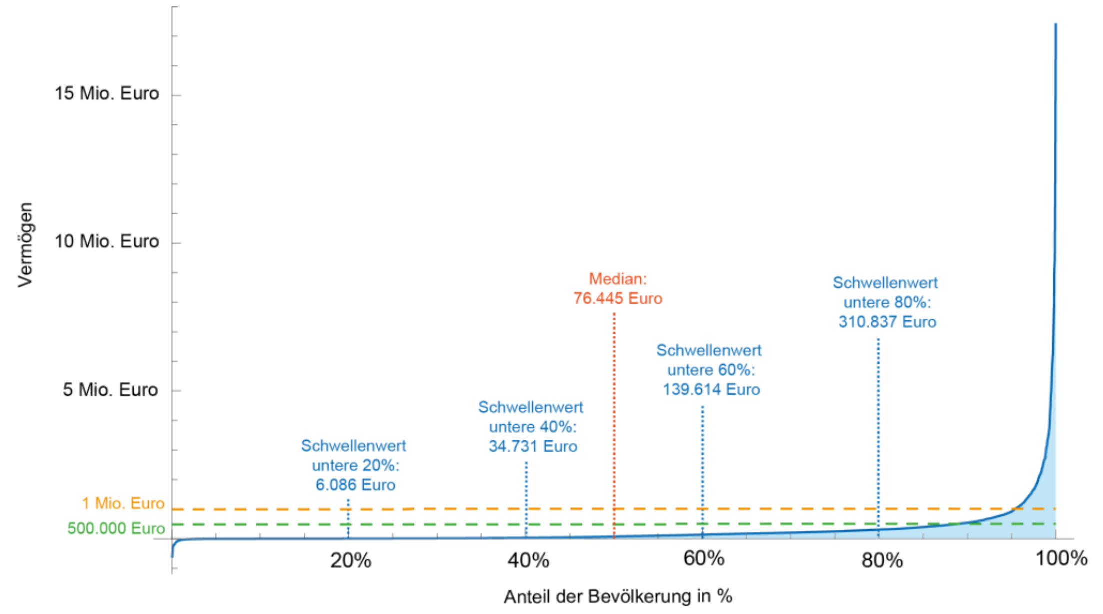

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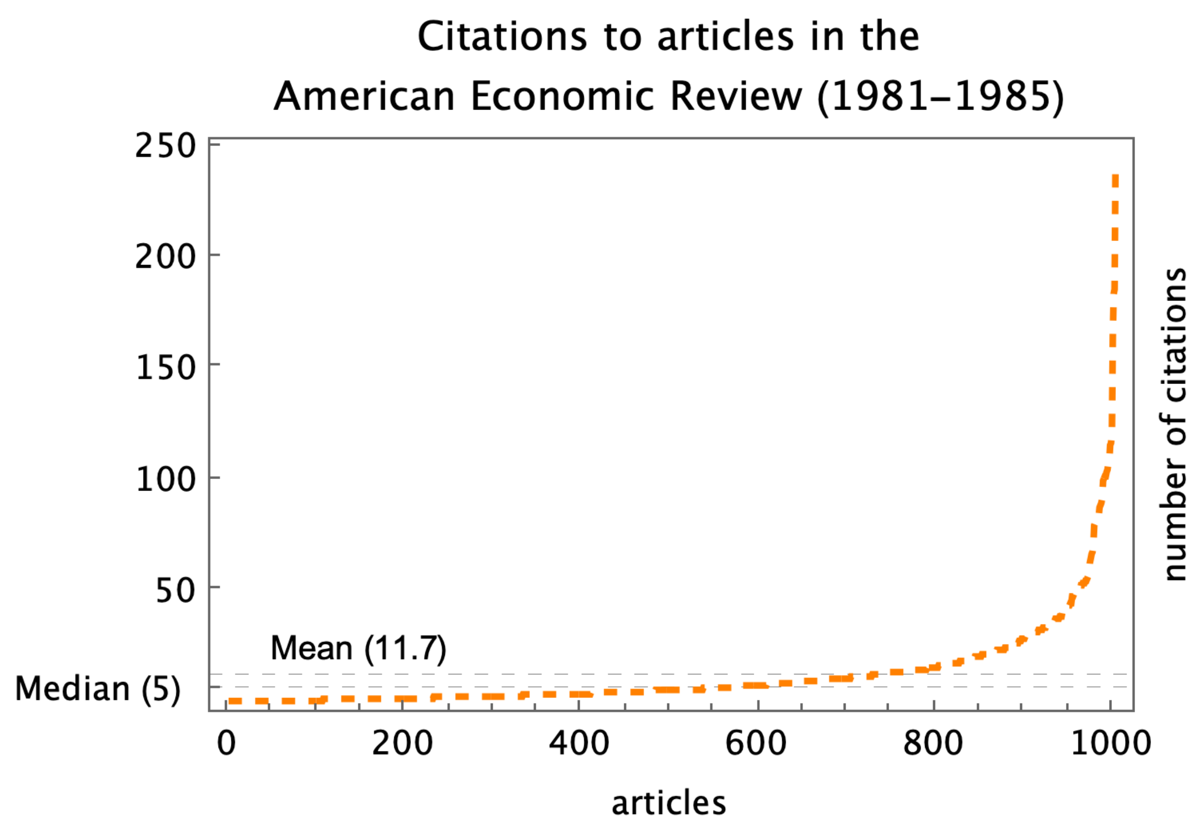

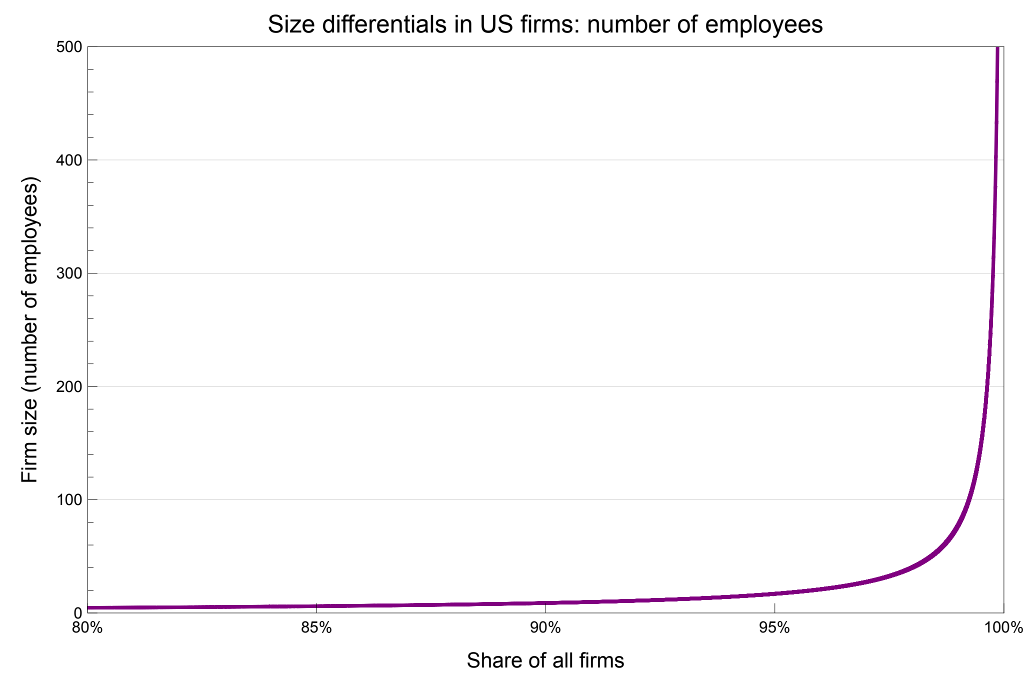

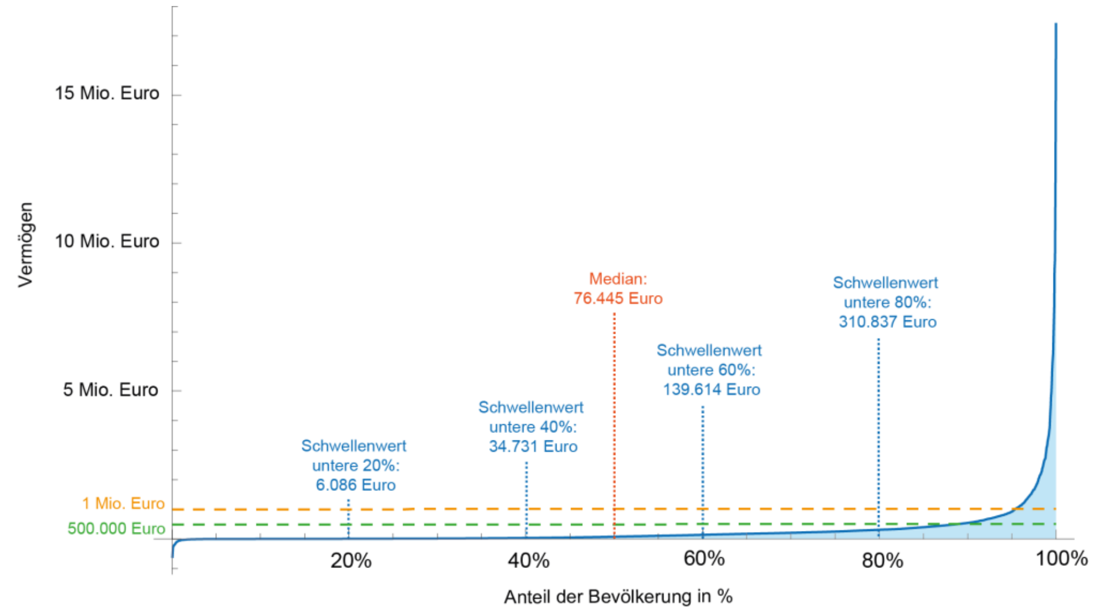

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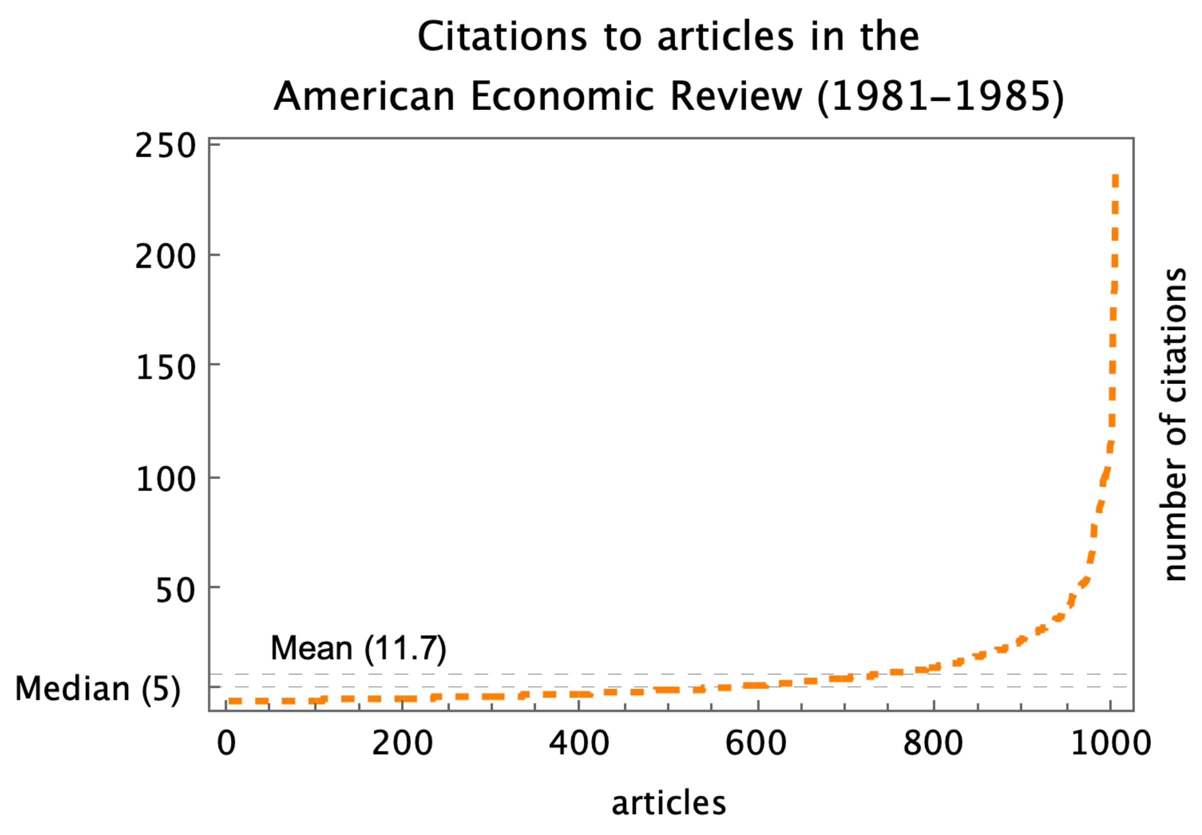

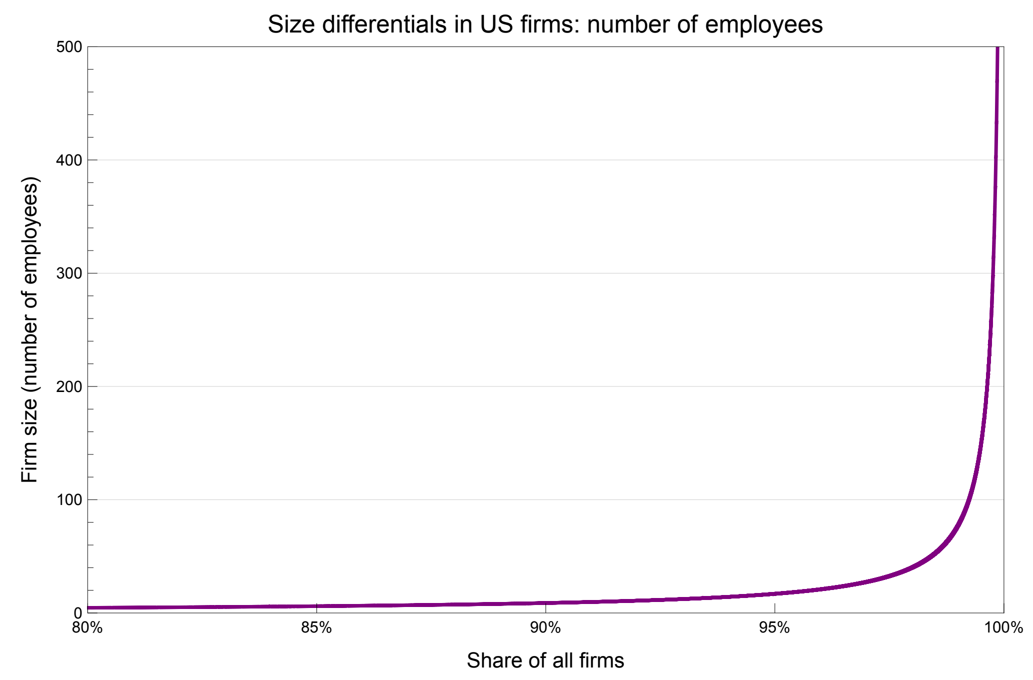

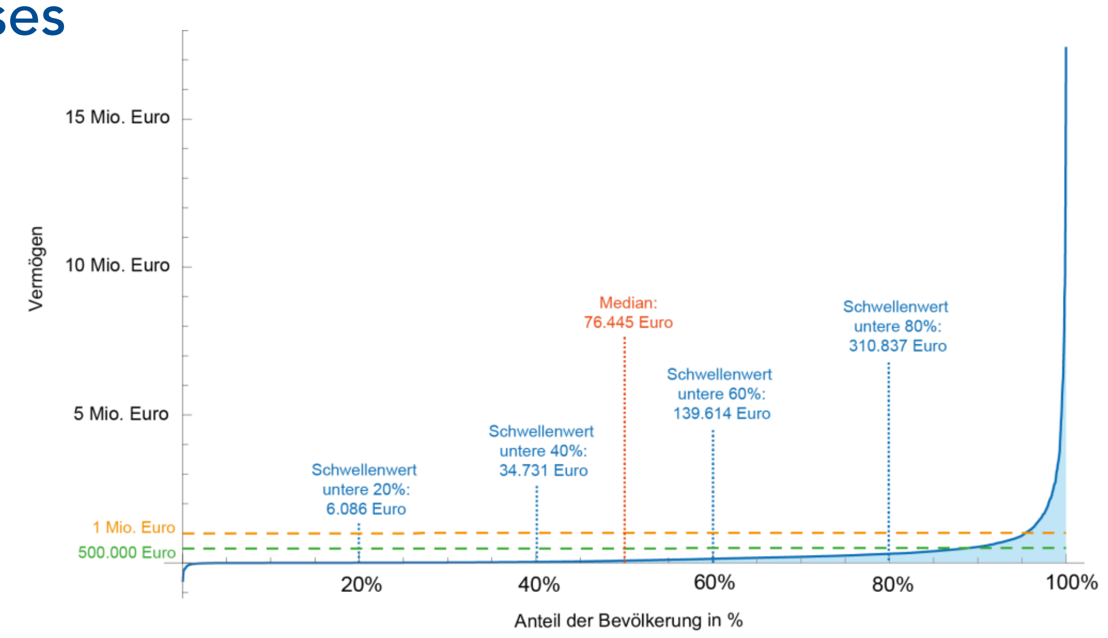

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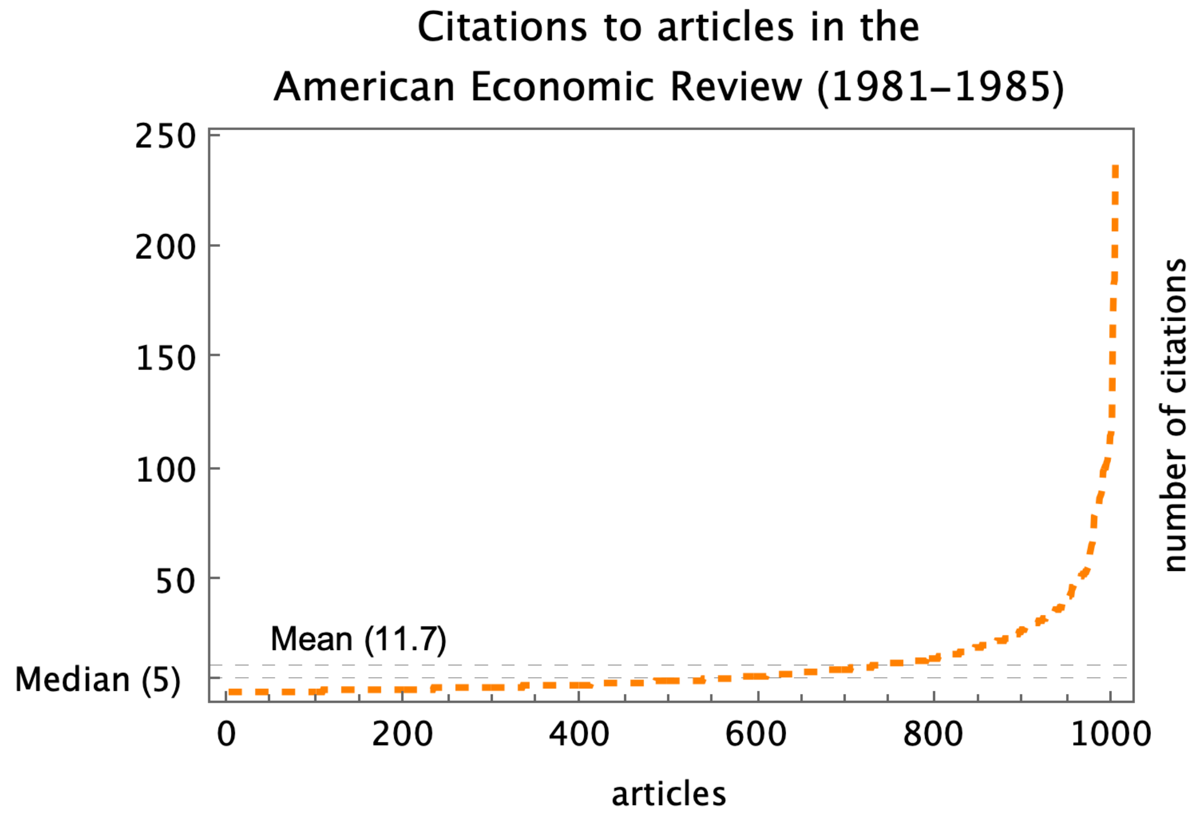

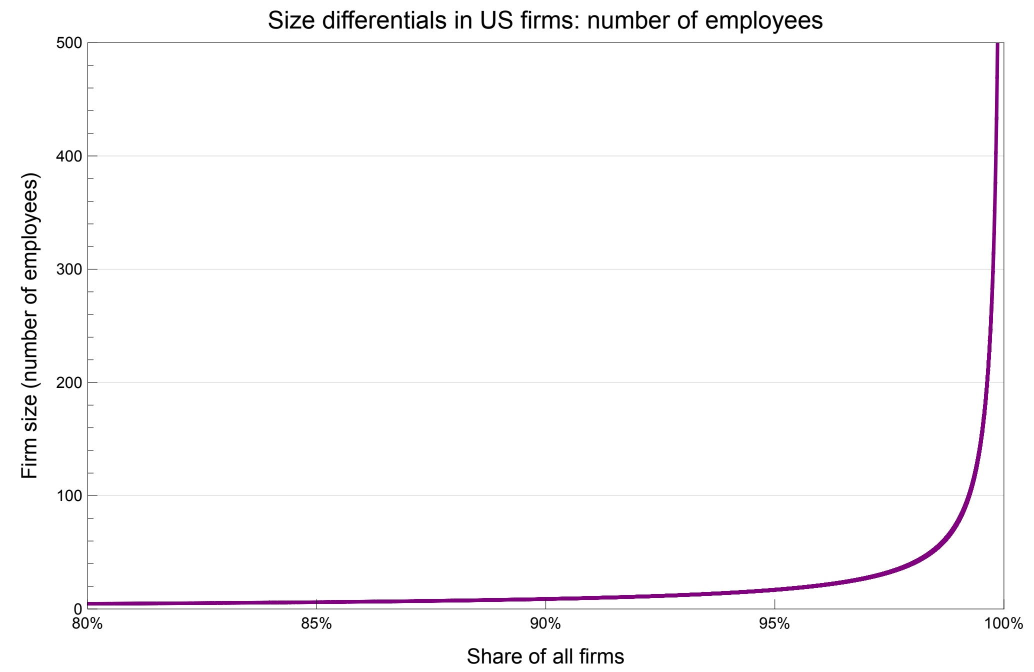

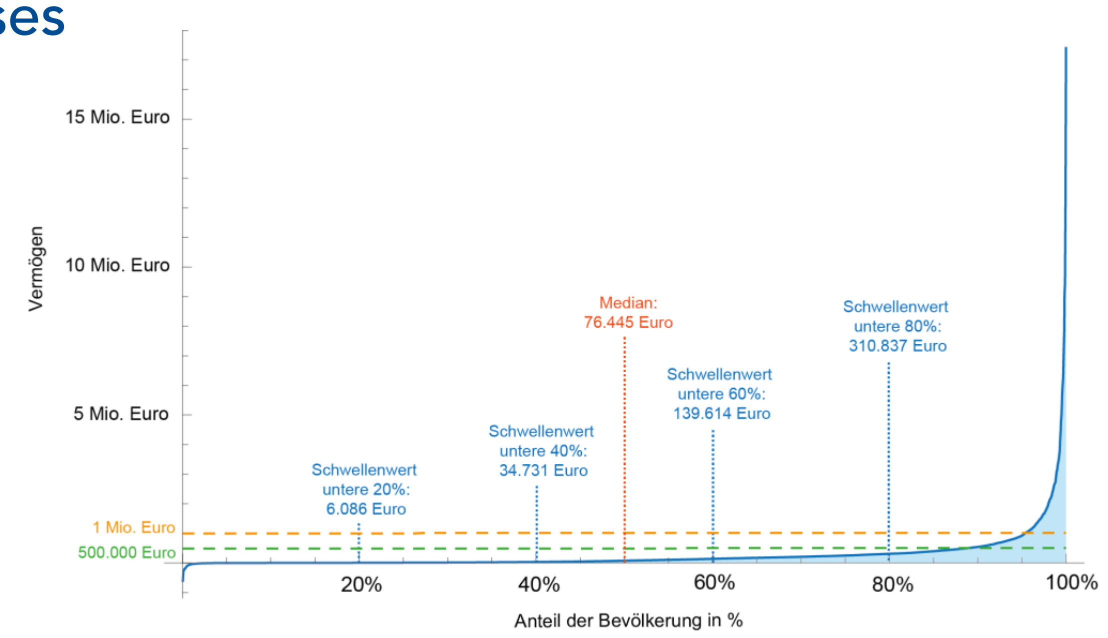

Cumulative advantage
(aka „Matthew effects“)

Cumulative advantage and Matthew effects

- Cumulative advantage as a plausible candidate mechanism for many empirically observed power law distributions
  - Cumulative advantage = „rich-get-richer“ dynamics = Matthew effects
  - Classic sources: talent (mainstream), differential saving rates (Kaldor, Marx), differential rates of return (Piketty, Shaikh, Veblen)

- There exists many models that follow such an intuition will generate power law distributions (under some conditions)…
  - Replicator dynamics (e.g. Stan Metcalfe)
  - Preferential attachment (e.g. Barabasi-Albert)
  - Technology adoption models with positive feedback (e.g. Brian Arthur)
  - Piketty's first law of capitalism in conjunction with $r > g$ and $s_\pi > s_w$ …

Newman (2005): Power Laws, Pareto Distributions and Zipf's Law.

Cumulative advantage and Matthew effects

- Cumulative advantage as a plausible candidate mechanism for many empirically observed power law distributions
  - Cumulative advantage = „rich-get-richer“ dynamics = Matthew effects
  - Classic sources: talent (mainstream), differential saving rates (Kaldor, Marx), differential rates of return (Piketty, Shaikh, Veblen)

- There exists many models that follow such an intuition will generate power law distributions (under some conditions)…
  - Replicator dynamics (e.g. Stan Metcalfe)
  - Preferential attachment (e.g. Barabasi-Albert)
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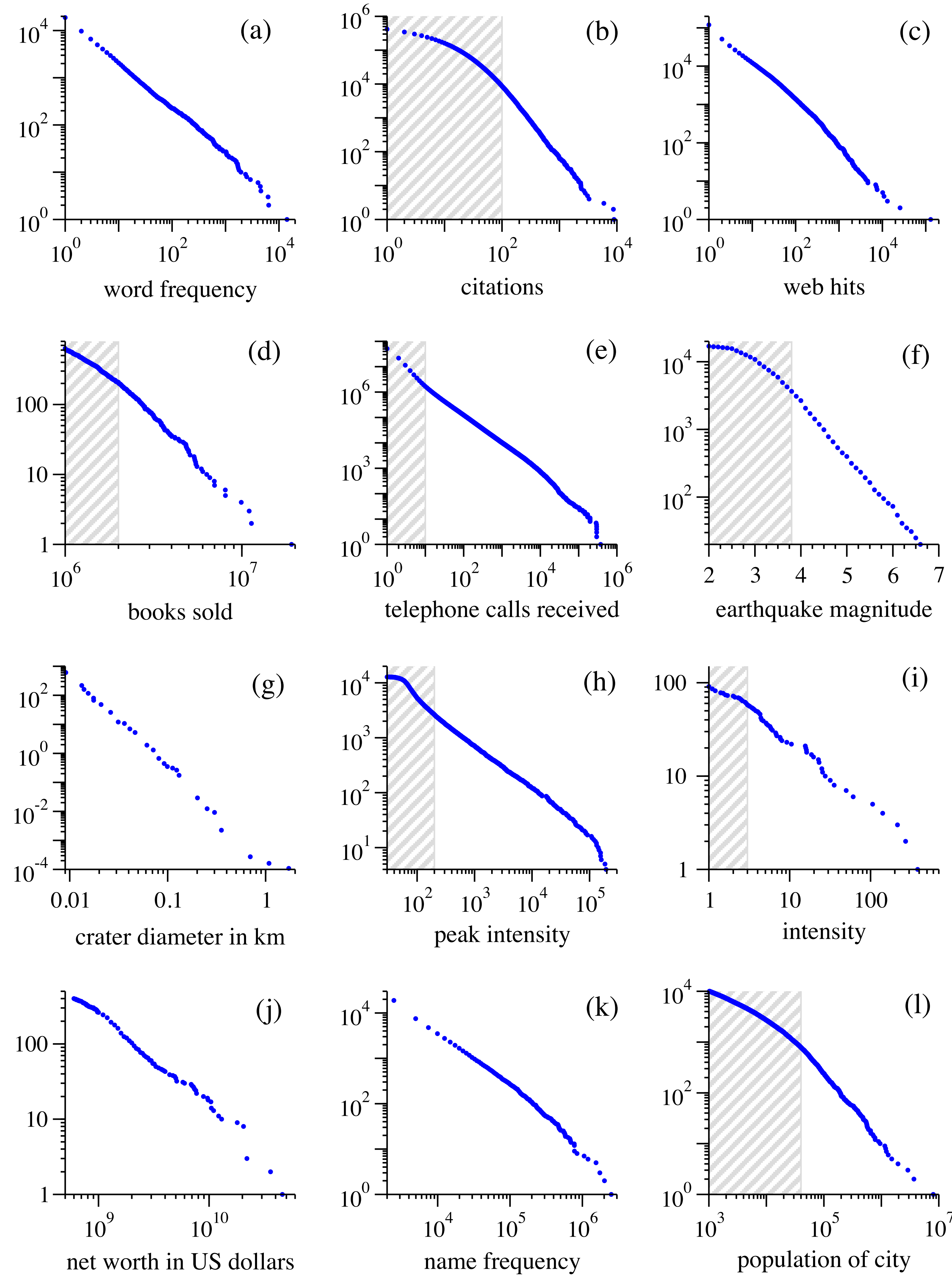

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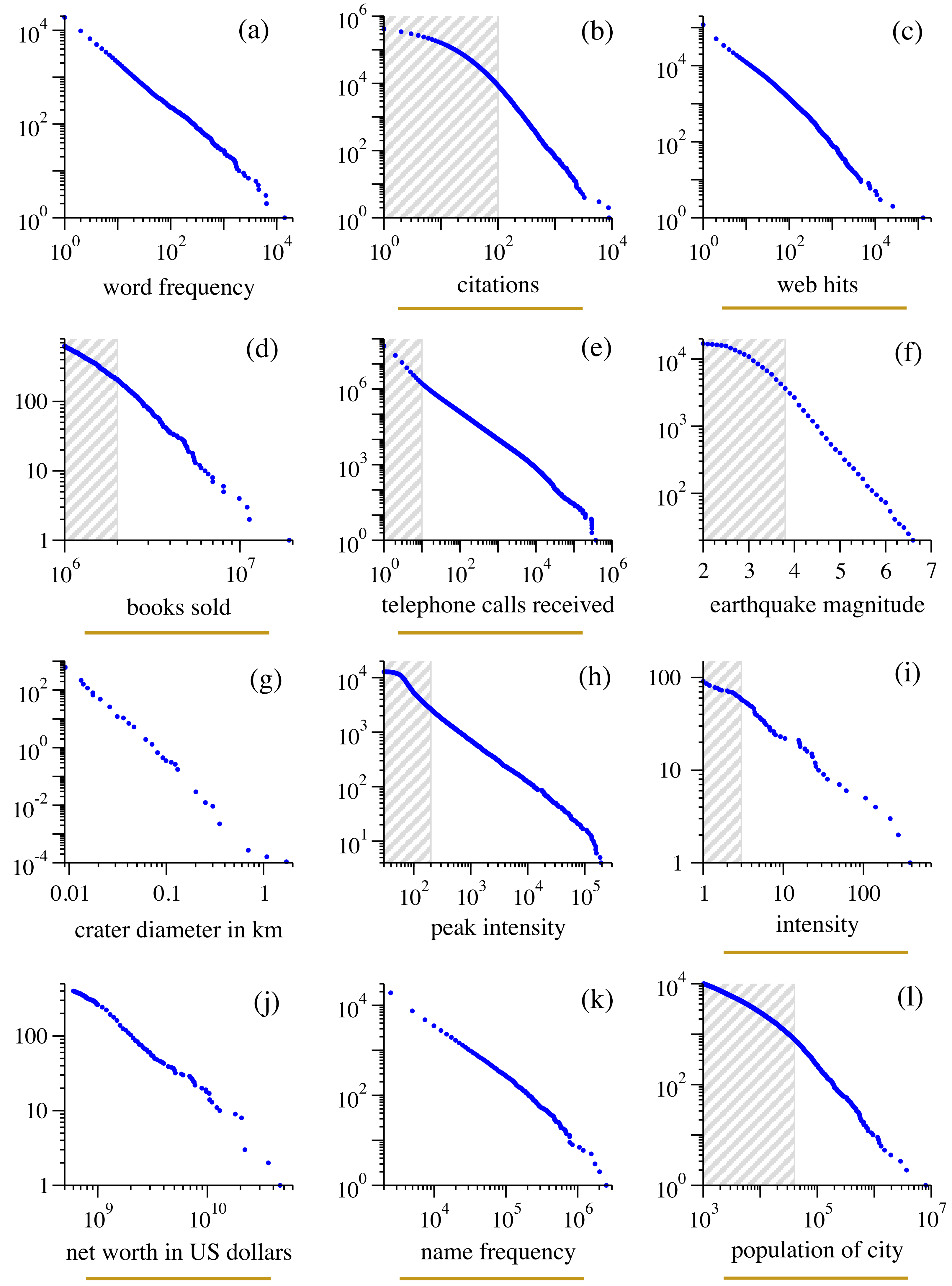

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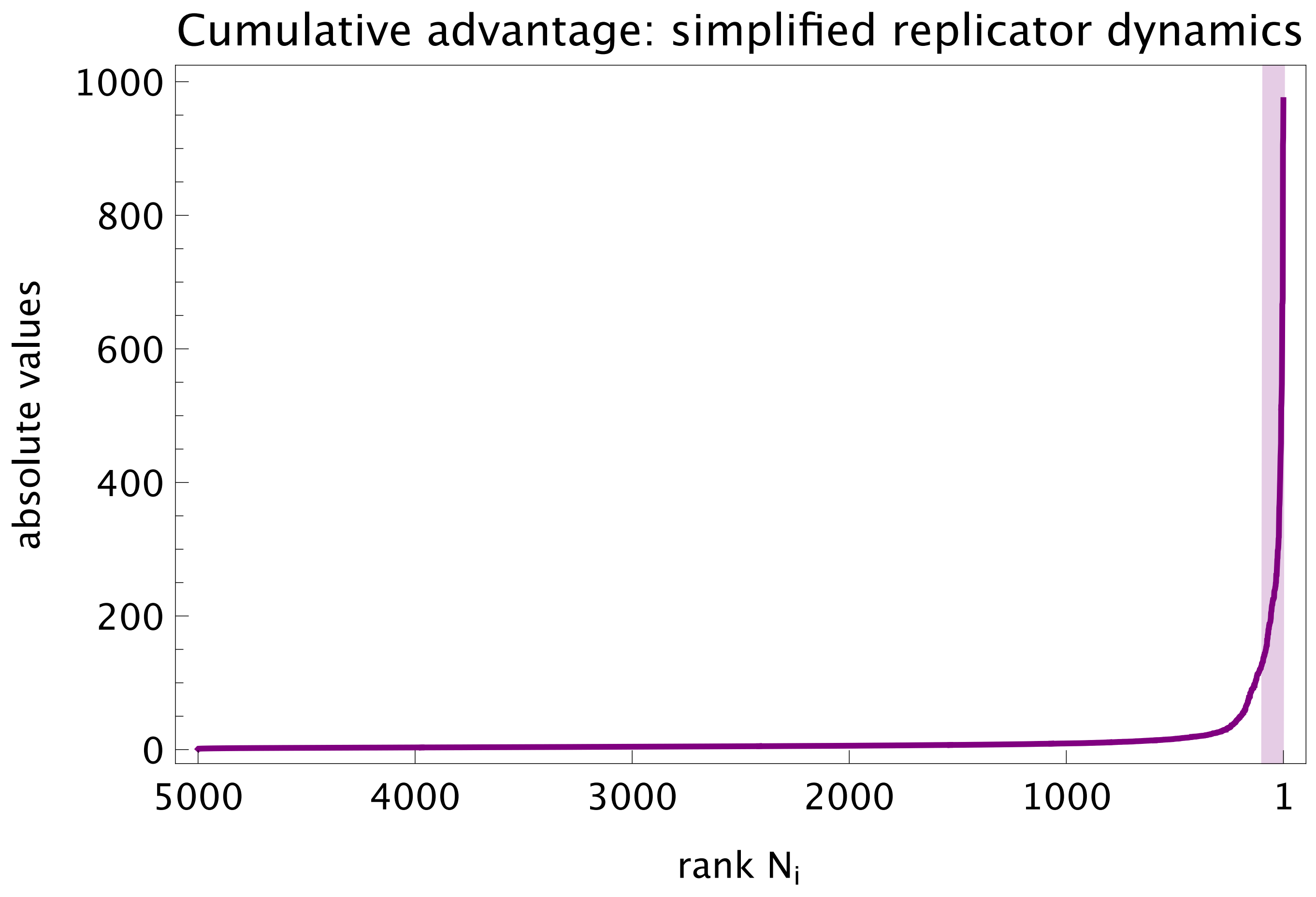
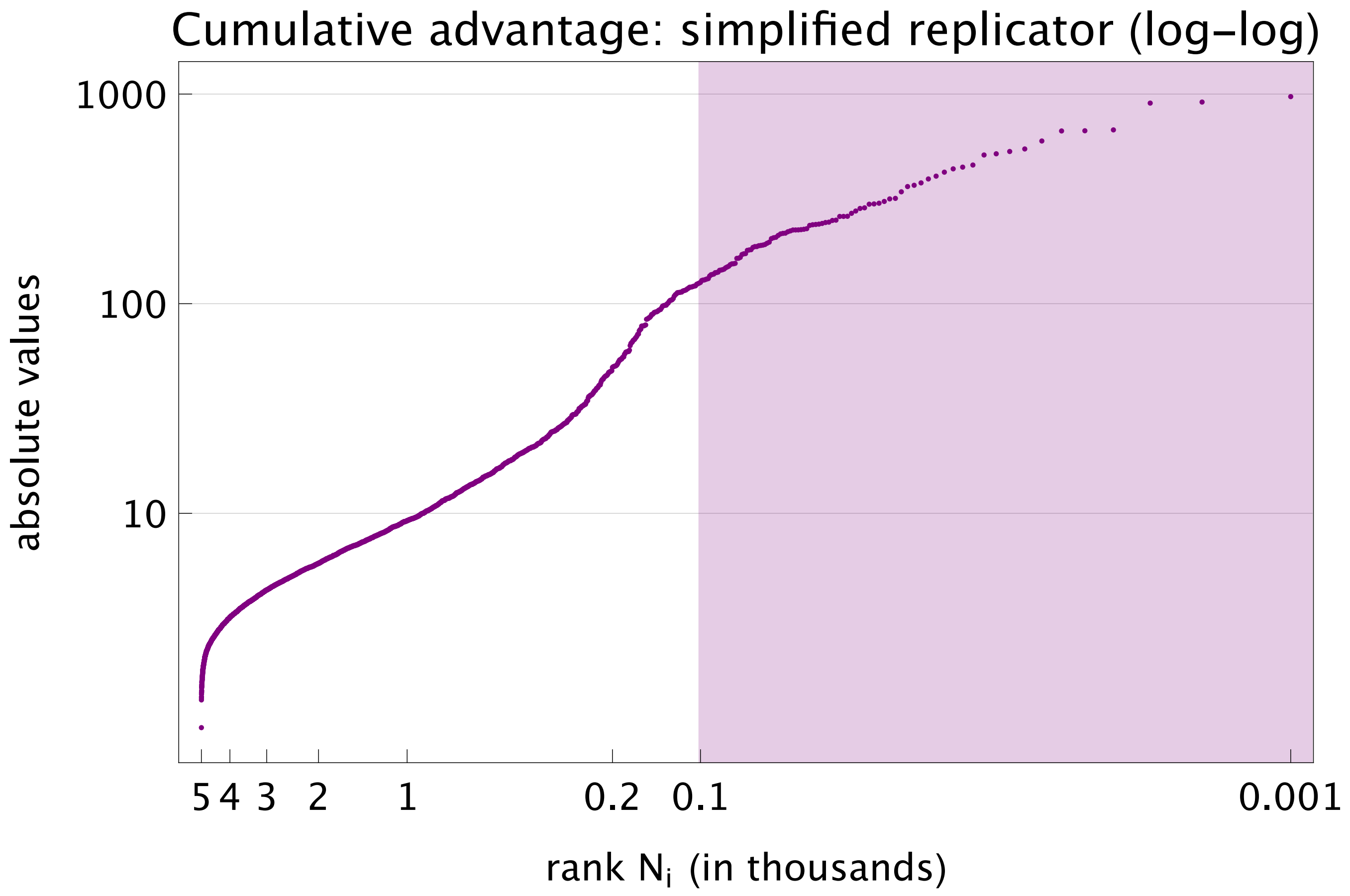
Very simple models of cumulative advantage

- Simplified replicator dynamics

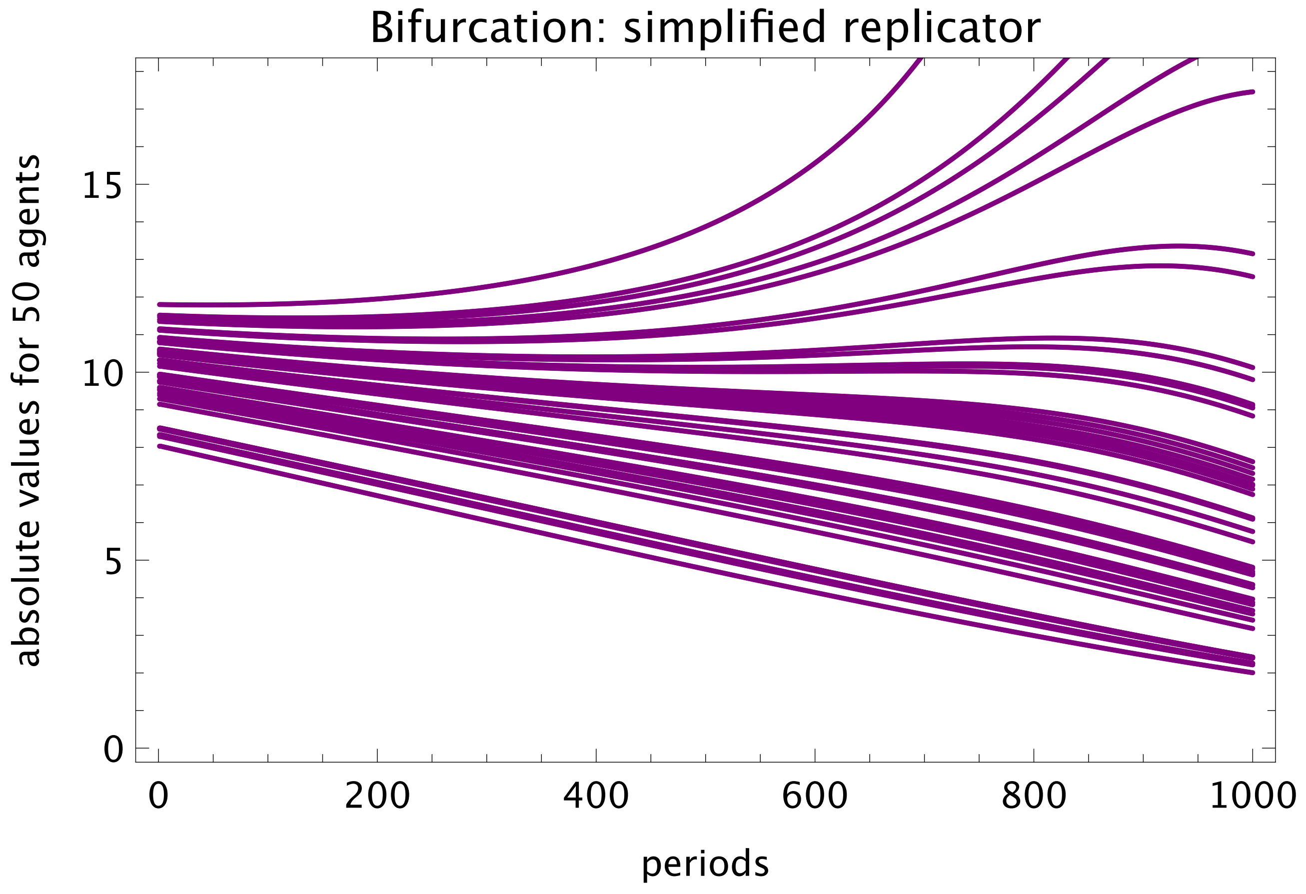
$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} \qquad \text{with} \qquad r_{i,t} = \left(\frac{w_{i,t}}{k \cdot \bar{w}} - 1\right)/m$$

  - Wealth dynamics with saving rate $s = 1$ and r that depends on ‚fitness', which depends on wealth vs. benchmark.





  - This leads to a bifurcation between rich and poor agents.
  - k can be used to regulate the bifurcation point, while m simply modulates the speed of the process (tractability).
  - Generates a power law at the top (and in parts around the center).
  - Needs some (initial) heterogeneity: starting values cluster around $w_0 \sim \mathcal{N}(10,1)$

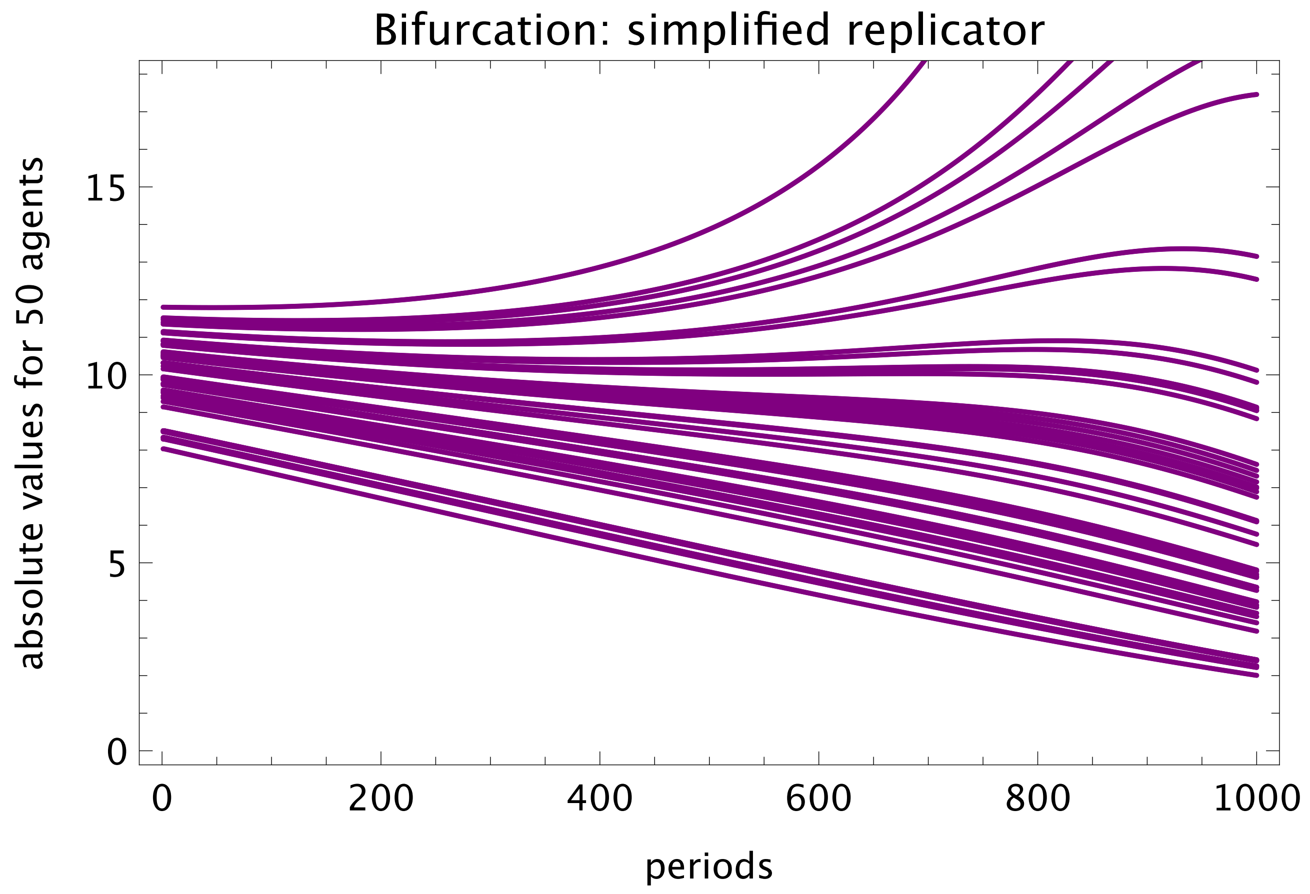


Very simple models of cumulative advantage

- ‚People have to eat' – model

  $w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} - c$ with $r_t \sim \mathcal{N}(\mu, \sigma)$

  - Intuition: additive anchor point should create similar bifurcation as wealth vs. benchmark.
  - But: additive component is fully exogenous, while individual fitness was not.



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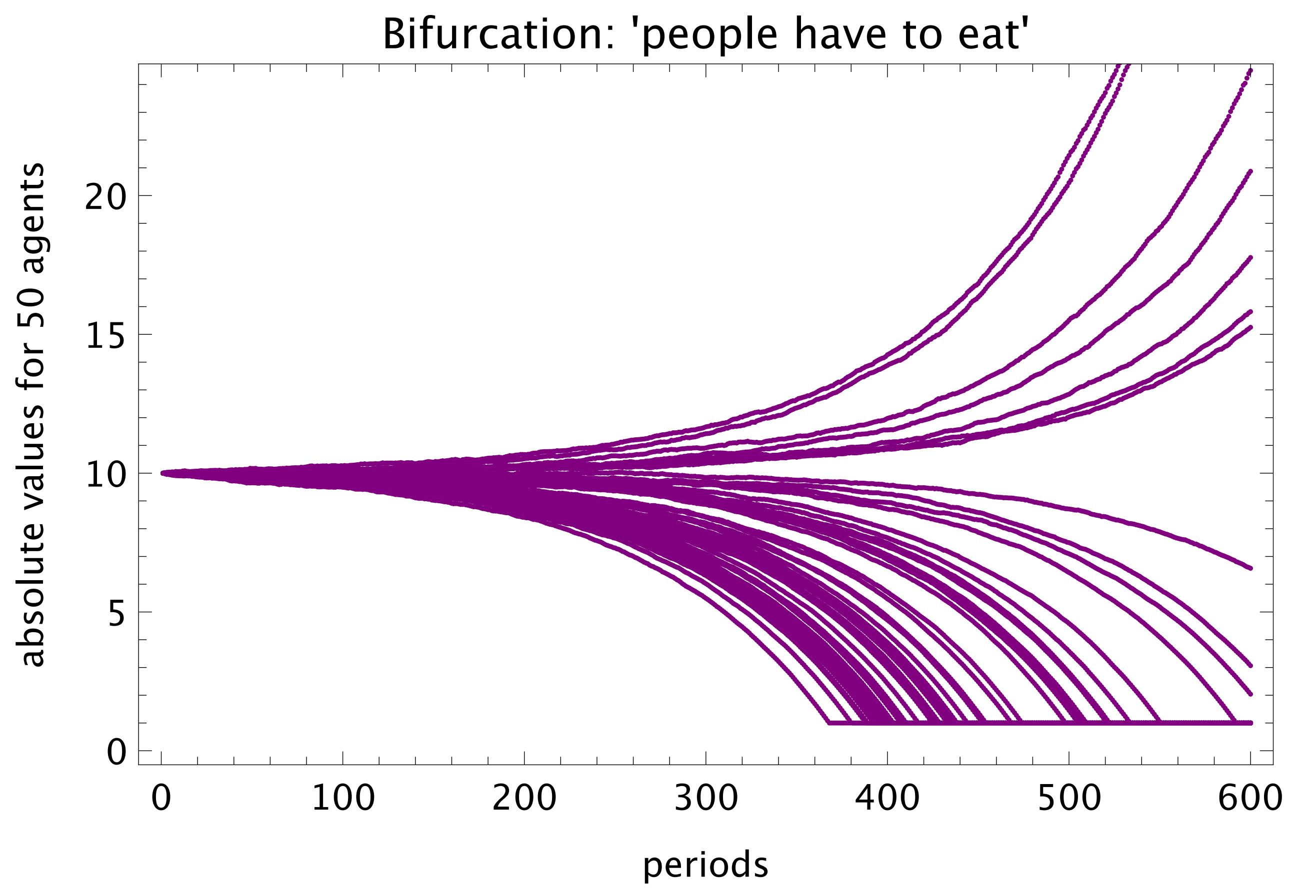

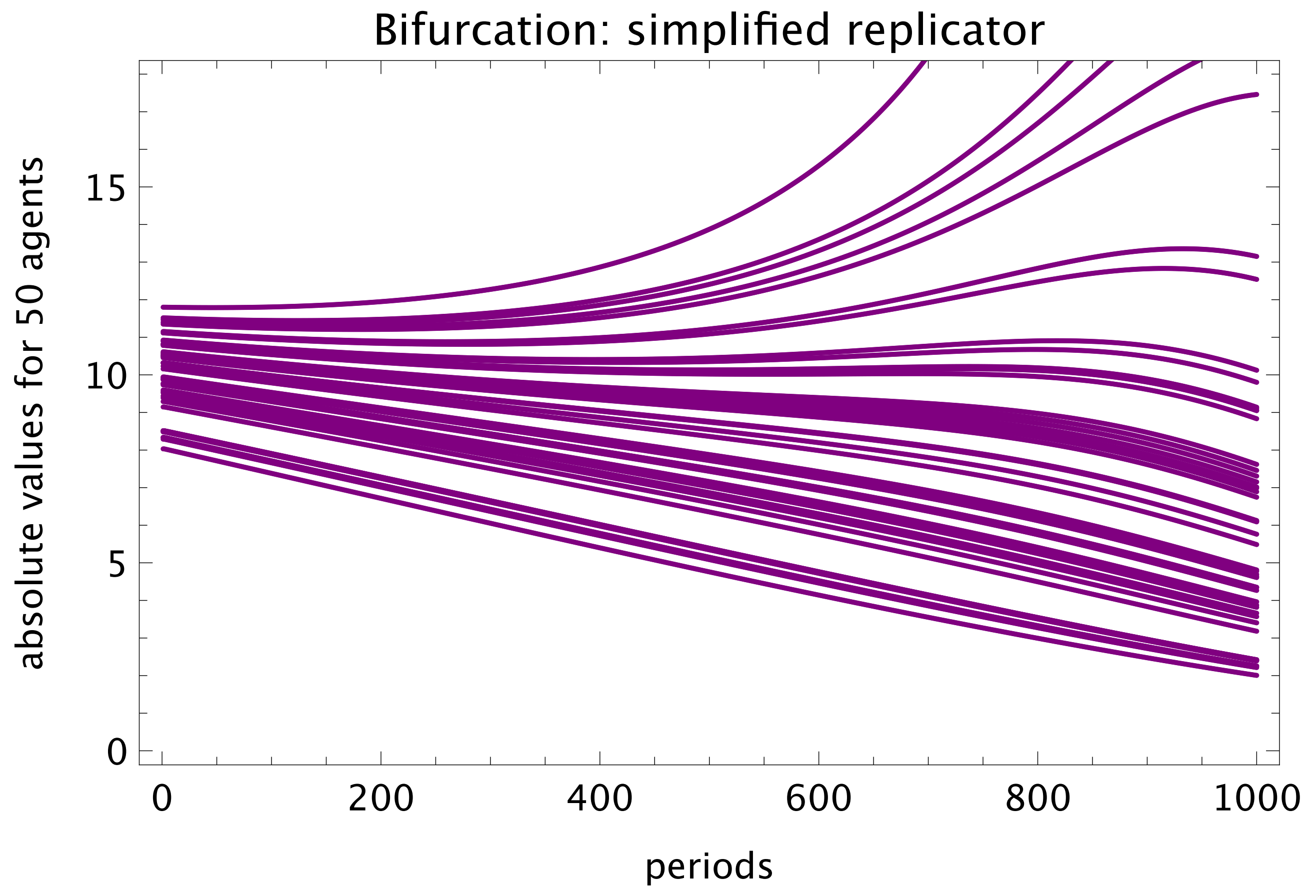

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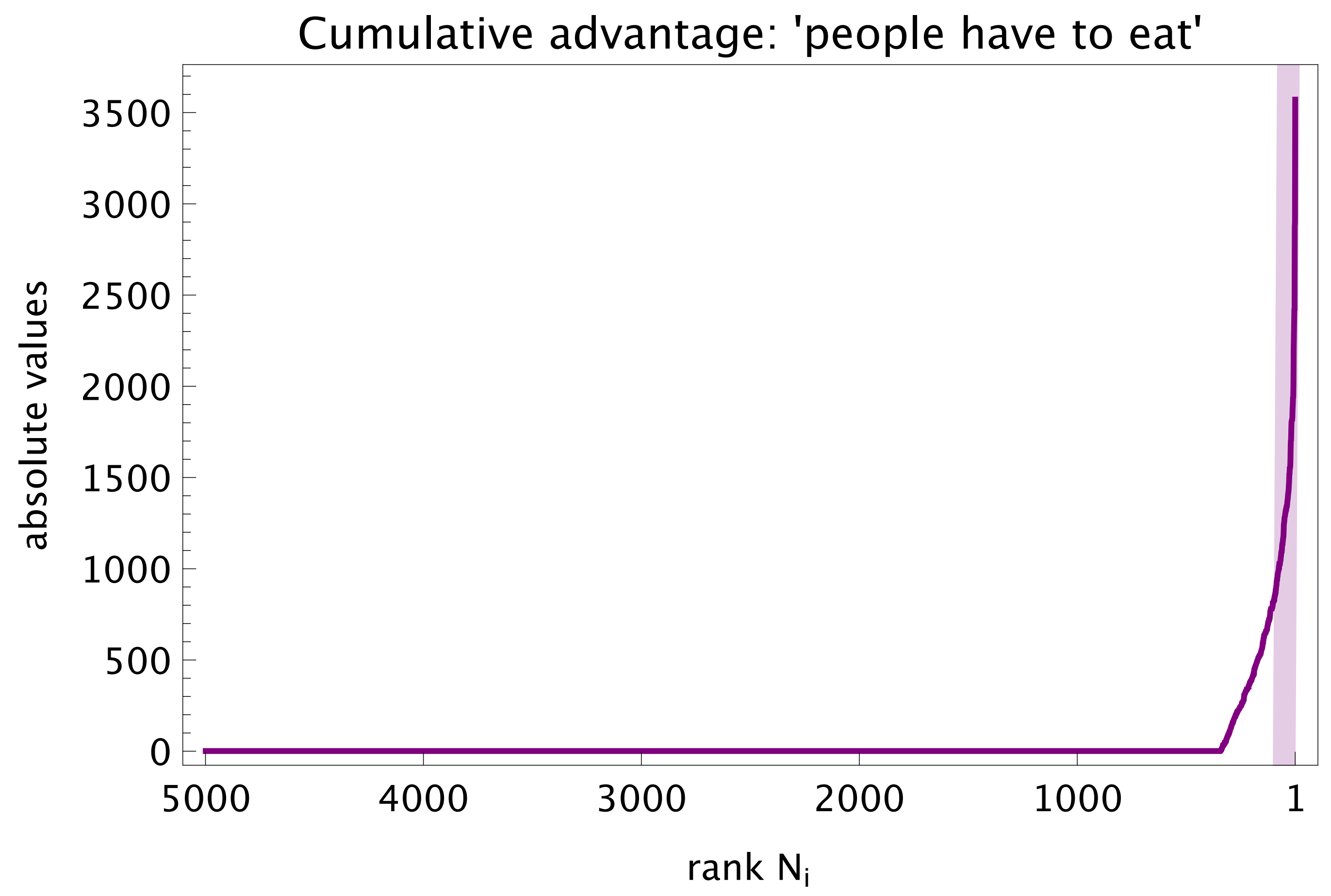

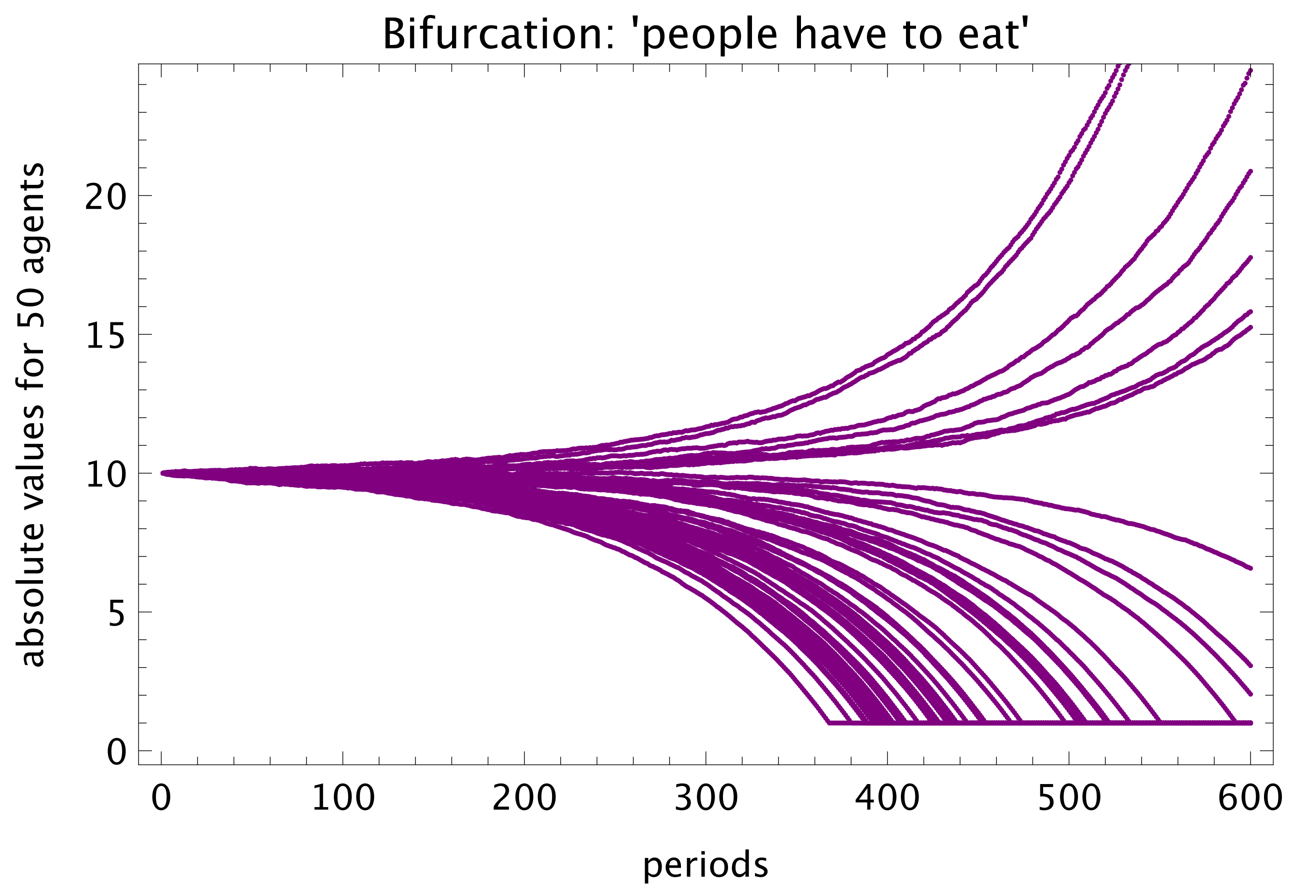

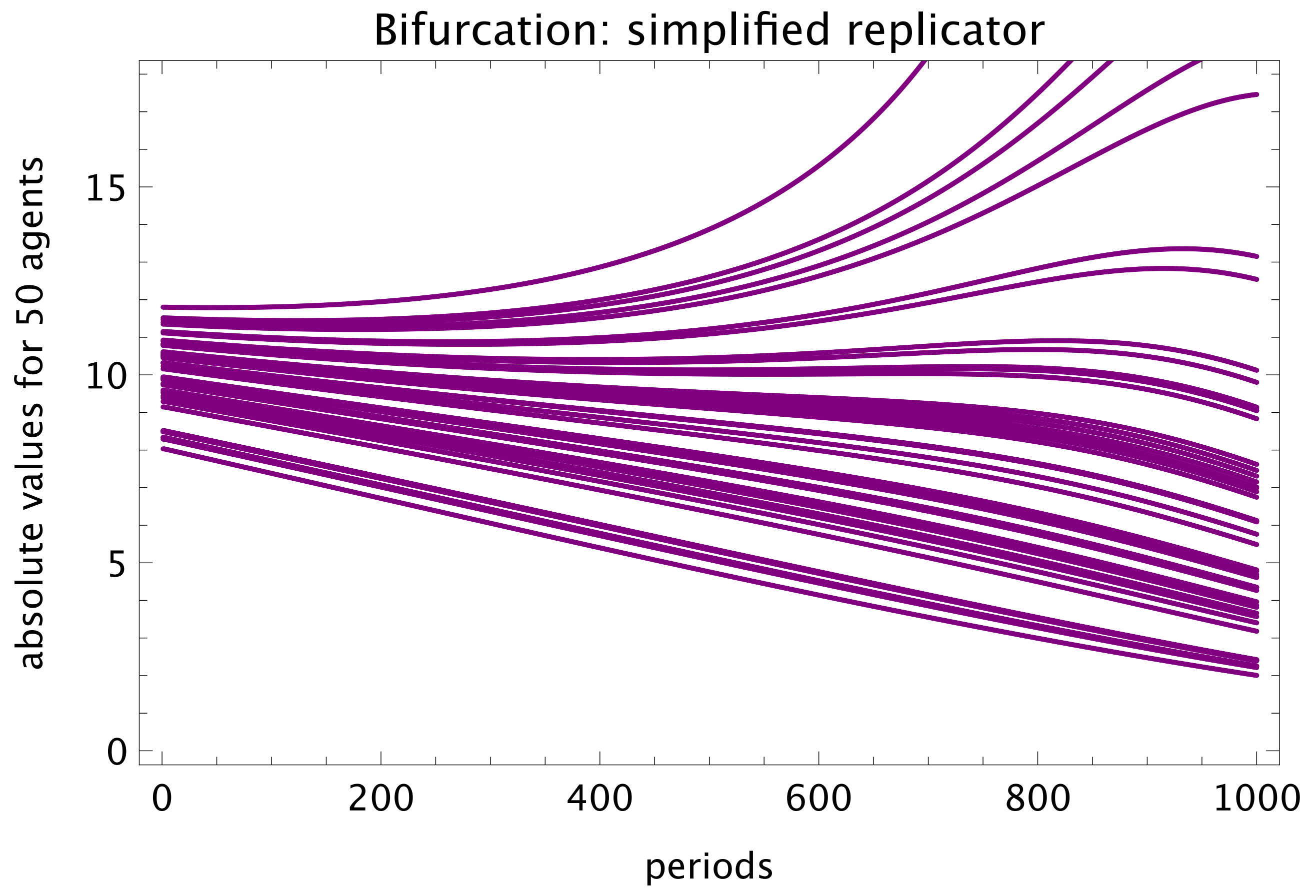

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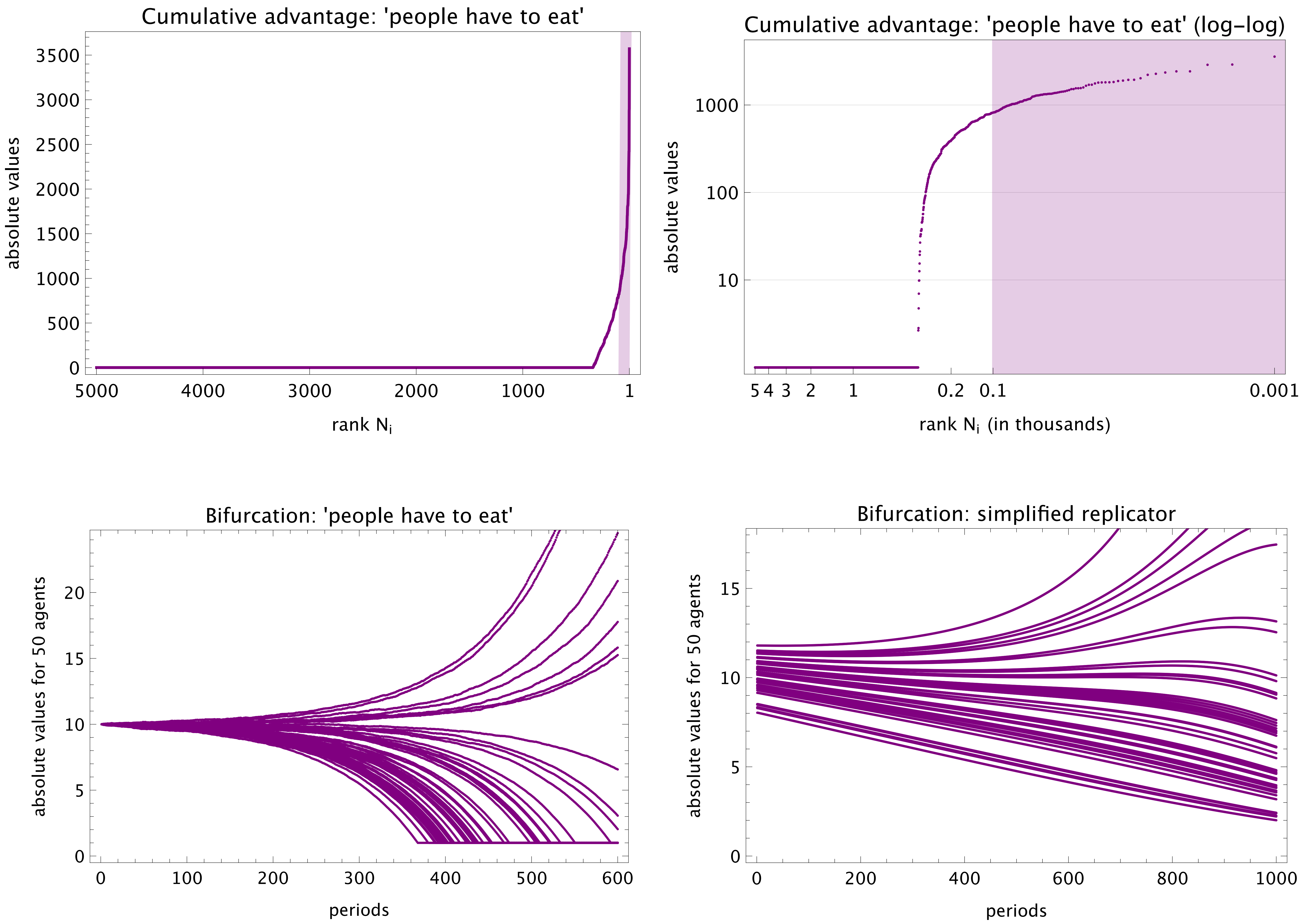

Jakob Kapeller

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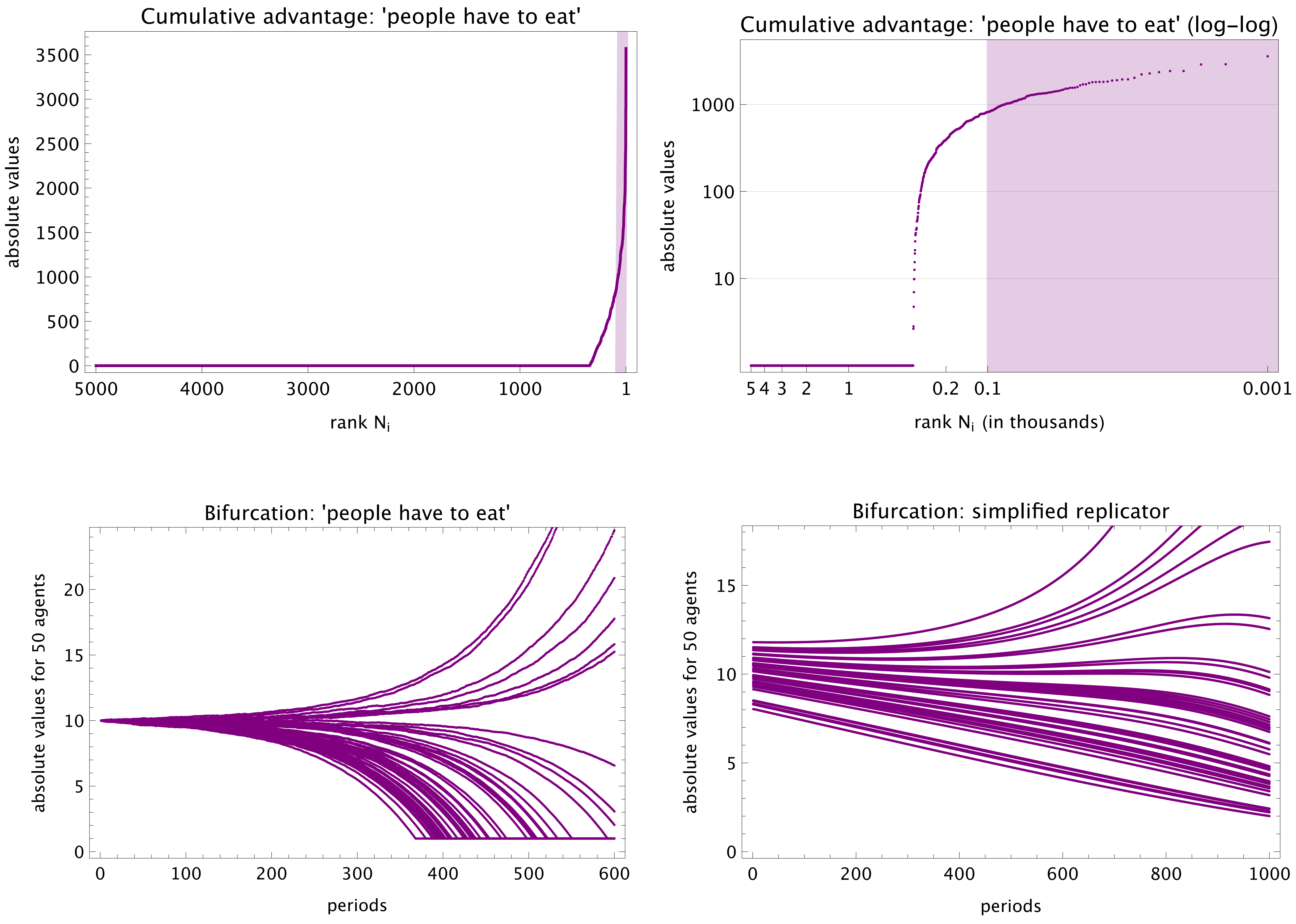
  - By defining $c = \mu \cdot w_0 + k$, we can again use k to regulate the bifurcation point.
  - Generates a power law at the top (and „saddle" in the center).
  - Also needs some heterogeneity (here: minor random element in growth rates)



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Very simple models of cumulative advantage (and the welfare state)

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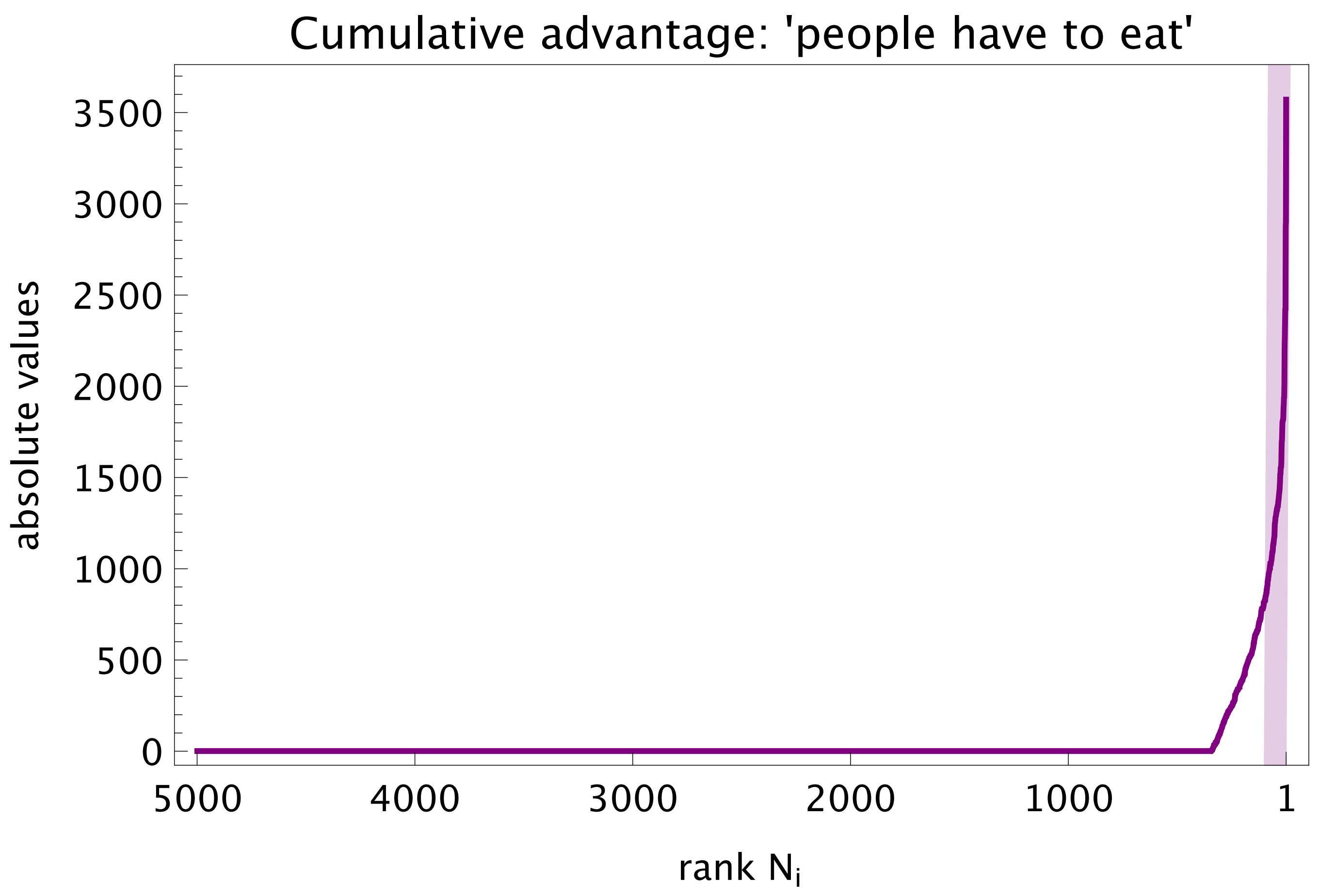
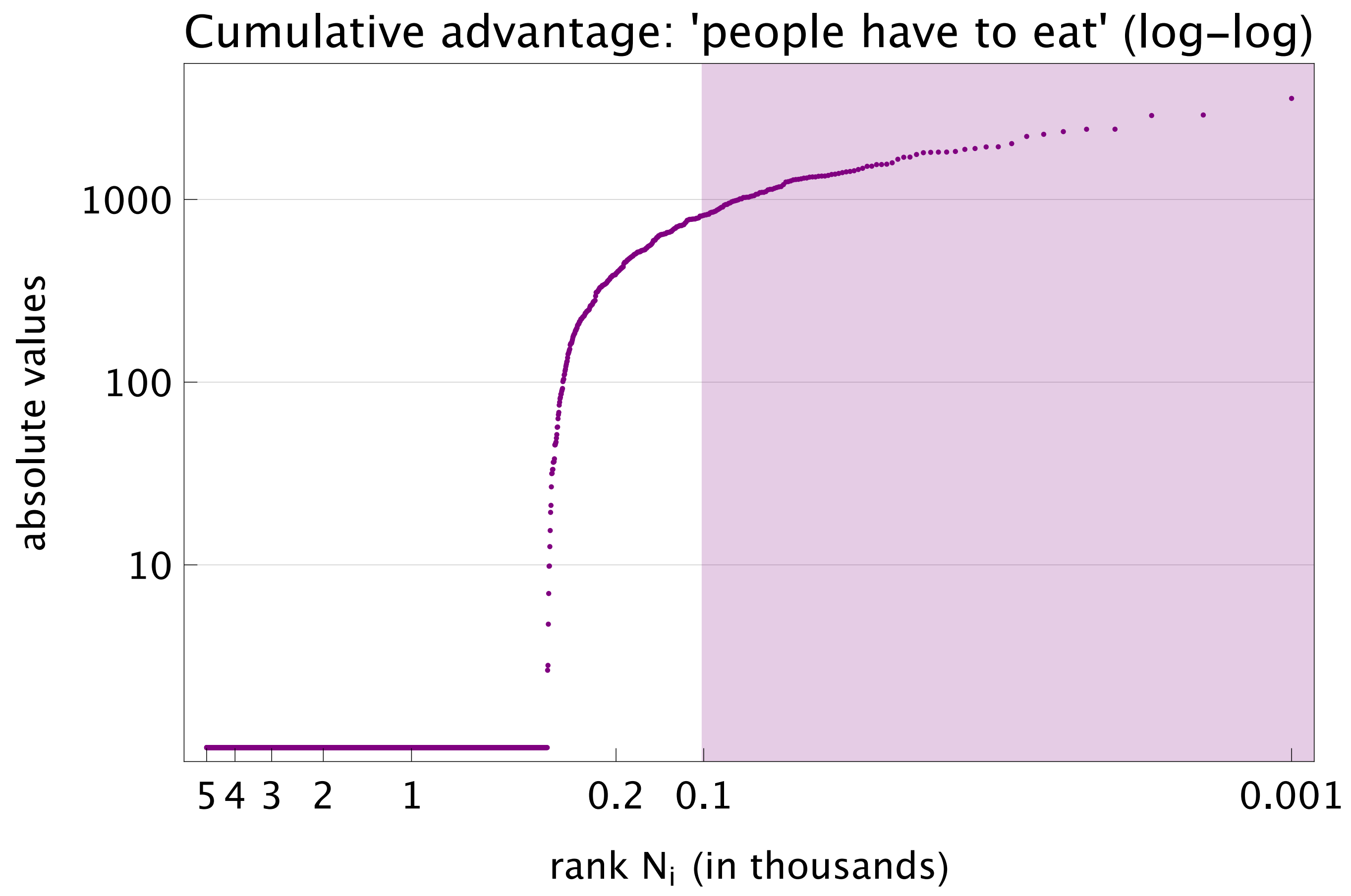
  - This would amount to a downward shift in k.





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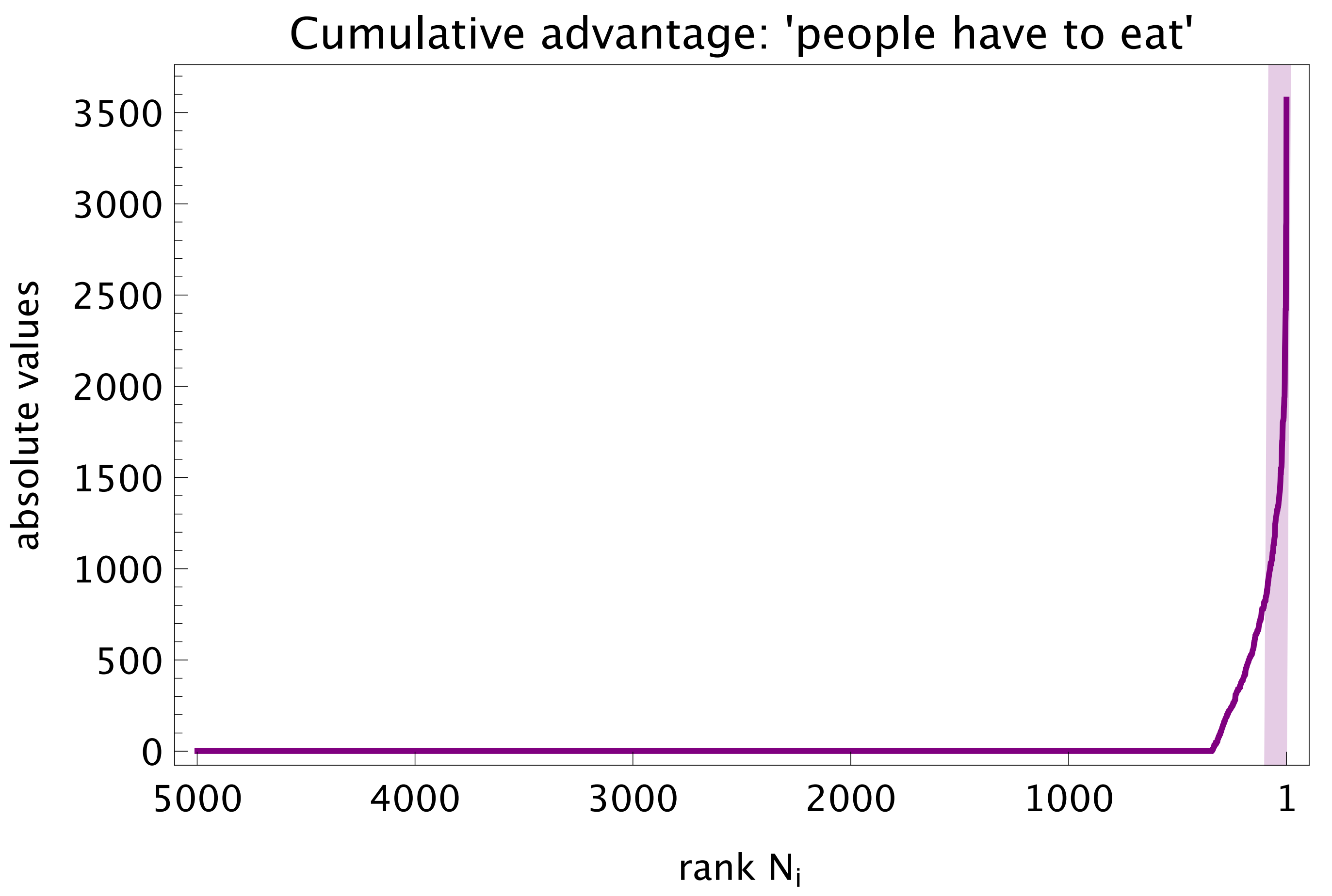

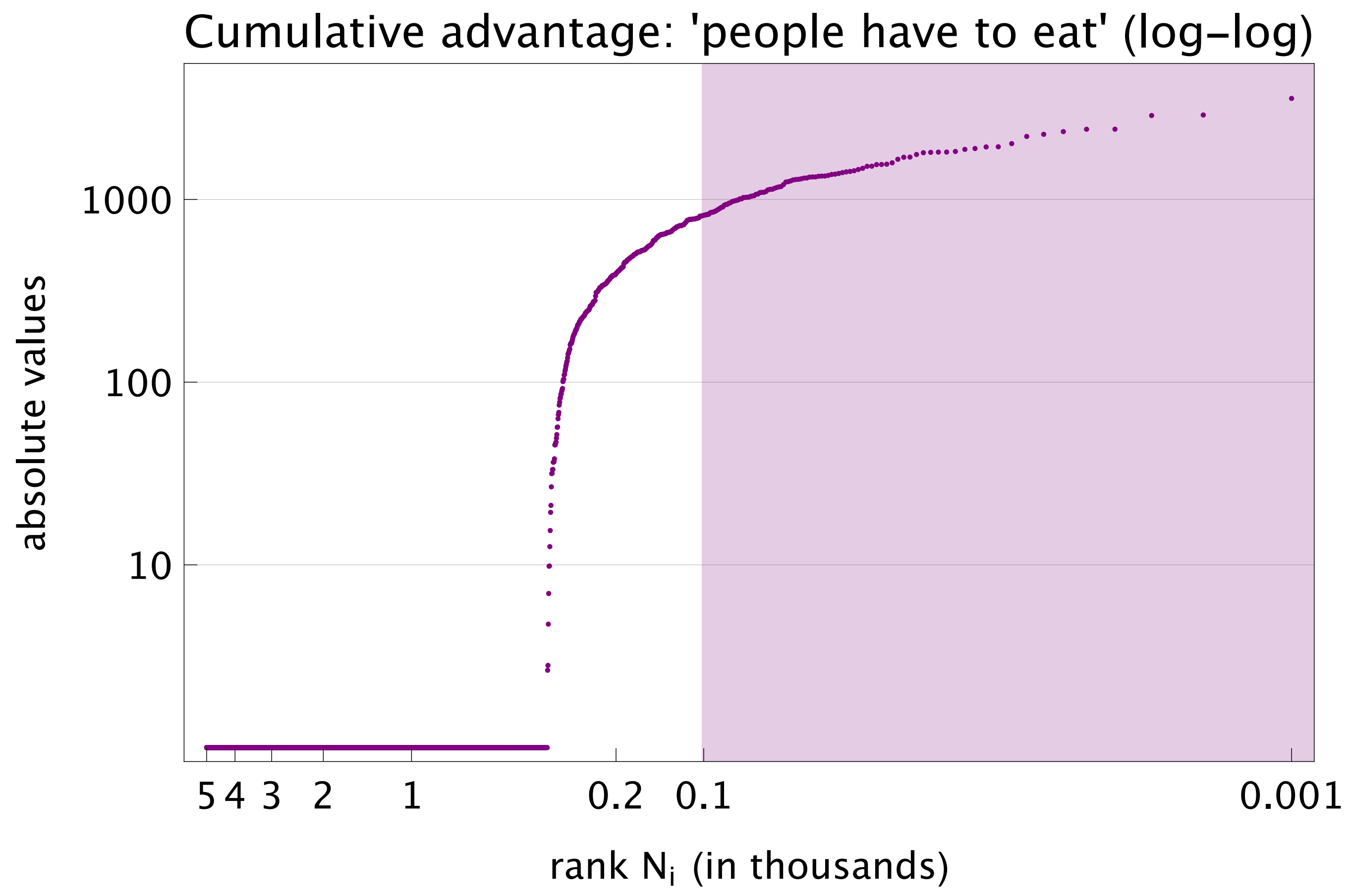

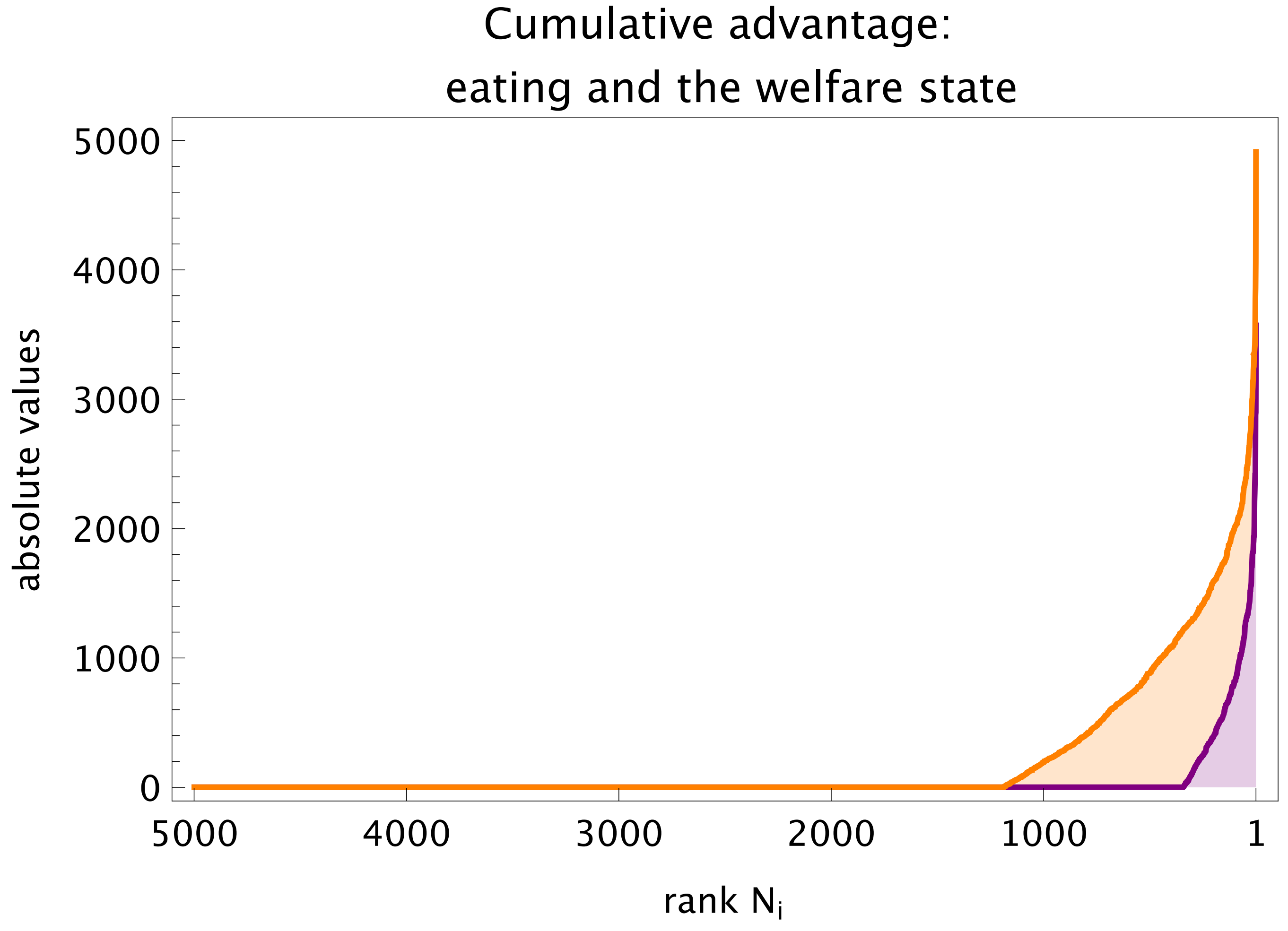

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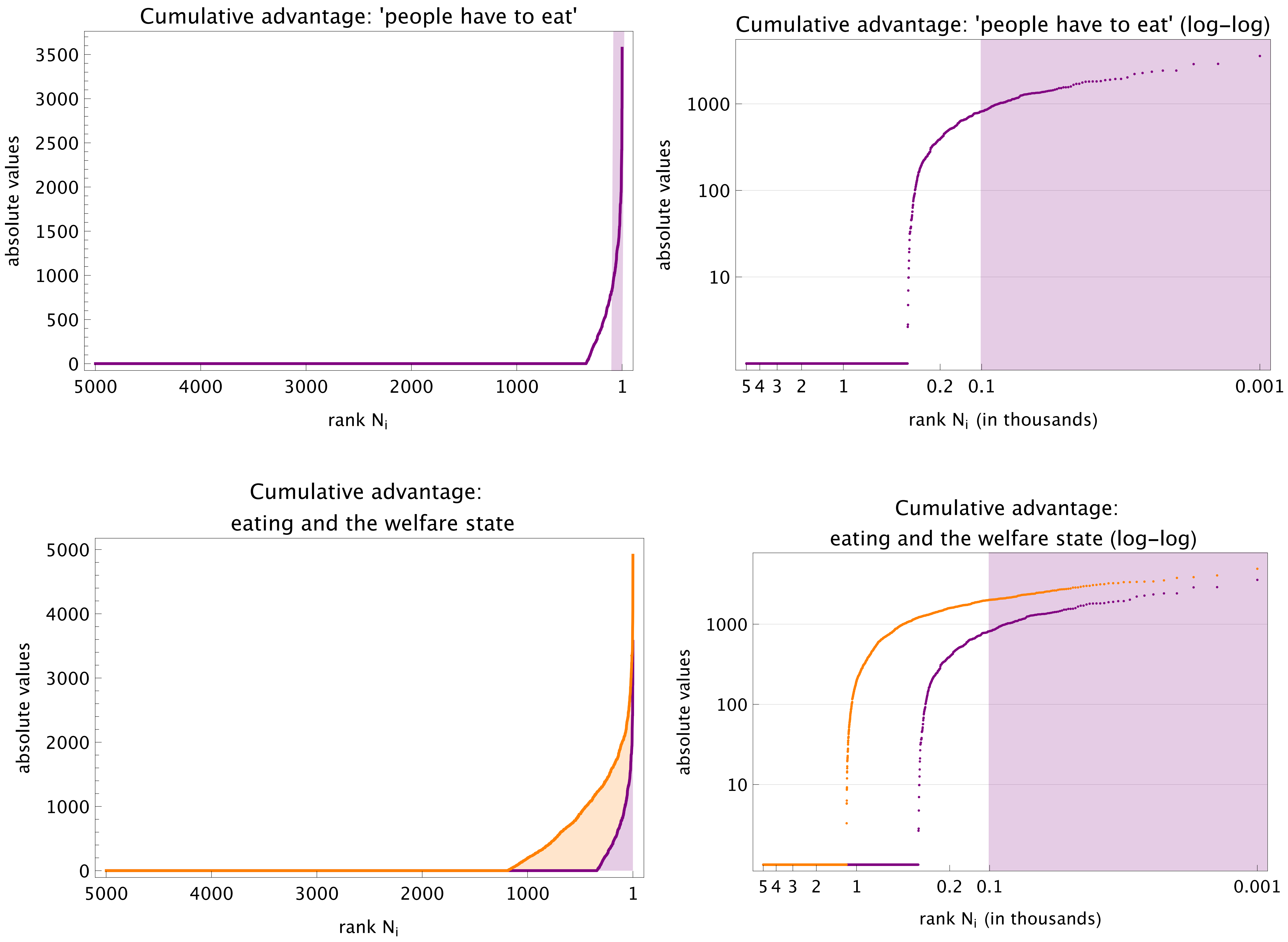

Cumulative advantage: some empirical traces

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Journal of Economic Literature 2021, 59(1), 3–44
https://doi.org/10.1257/jel.20191449

Ongoing concentration

Economic Inequality in Preindustrial Times: Europe and Beyond†

Guido Alfani*

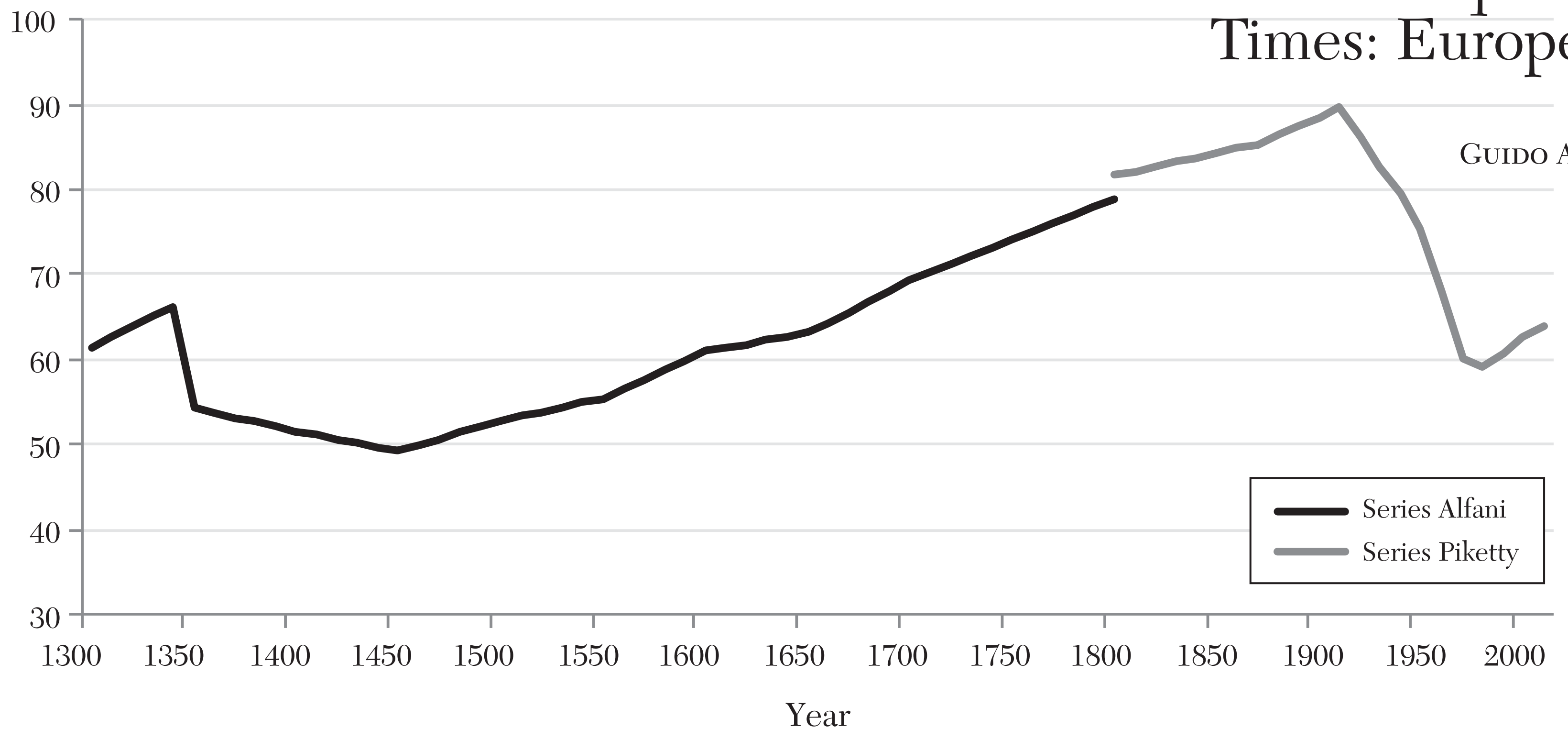
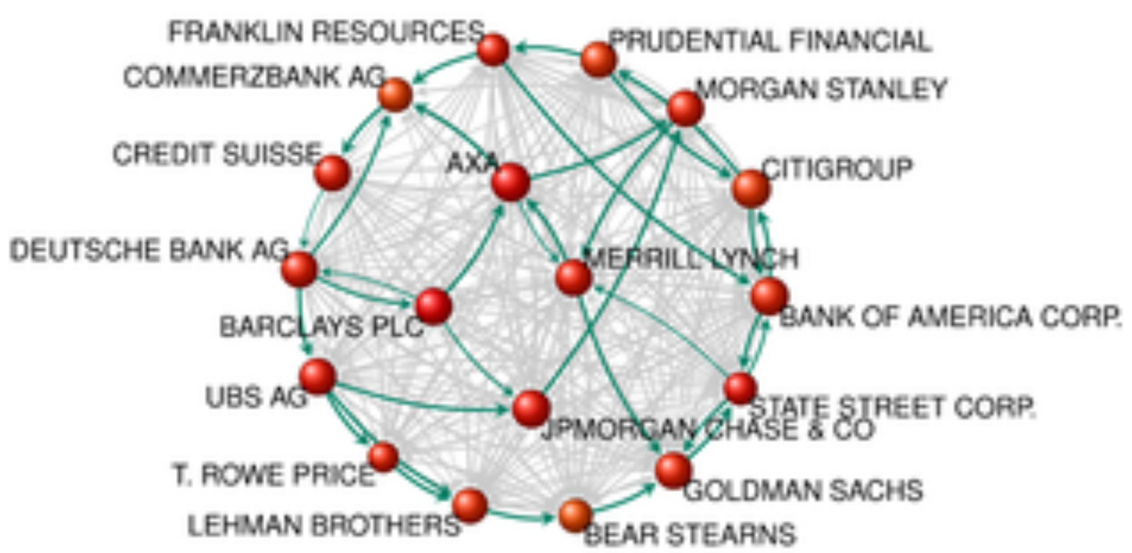


Figure 8. The Share of Wealth of the Top 10 Percent Rich in Europe, 1300–2010



Vitali/Glattfelder/Battiston (2011): The Network of Global Corporate Control. PLOSOne.
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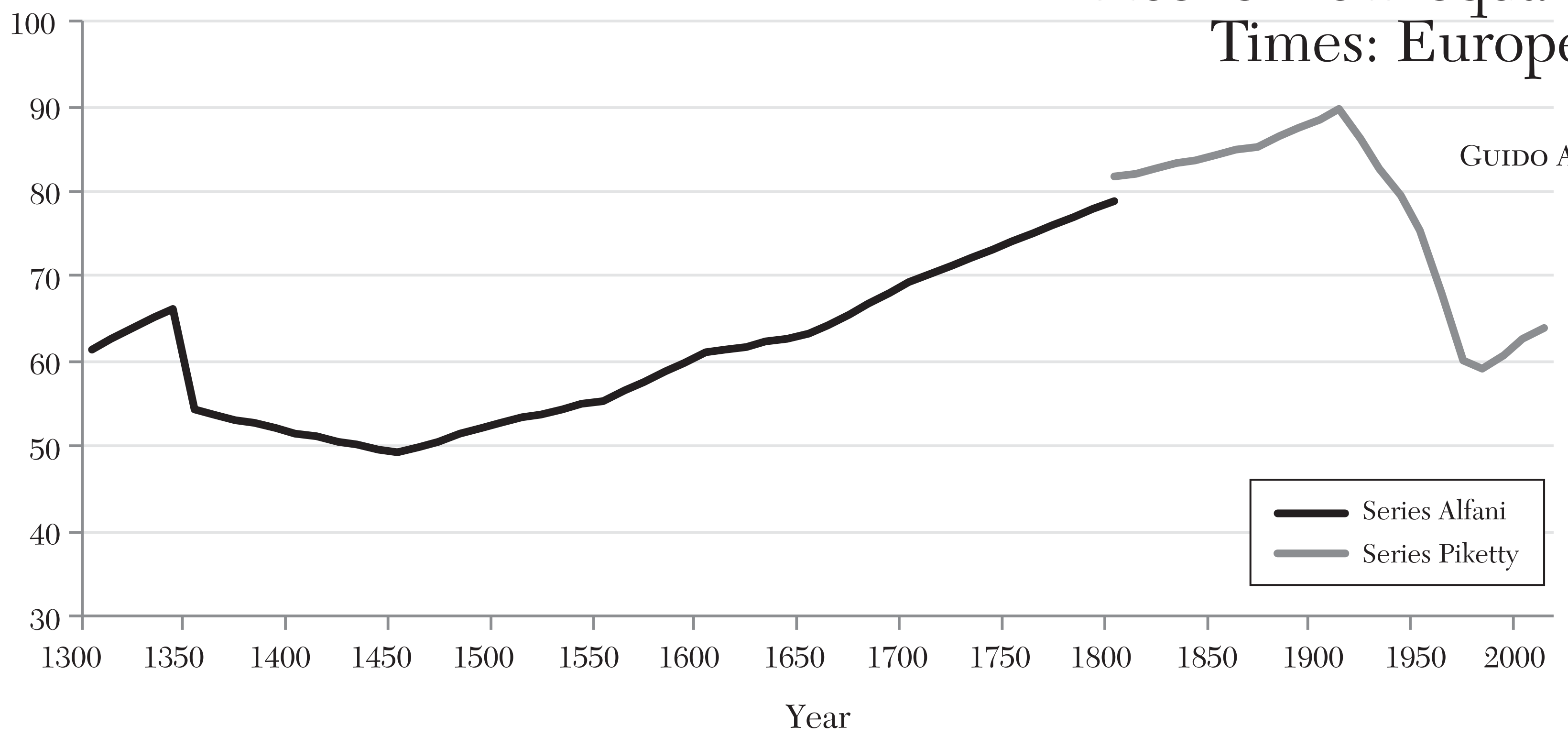

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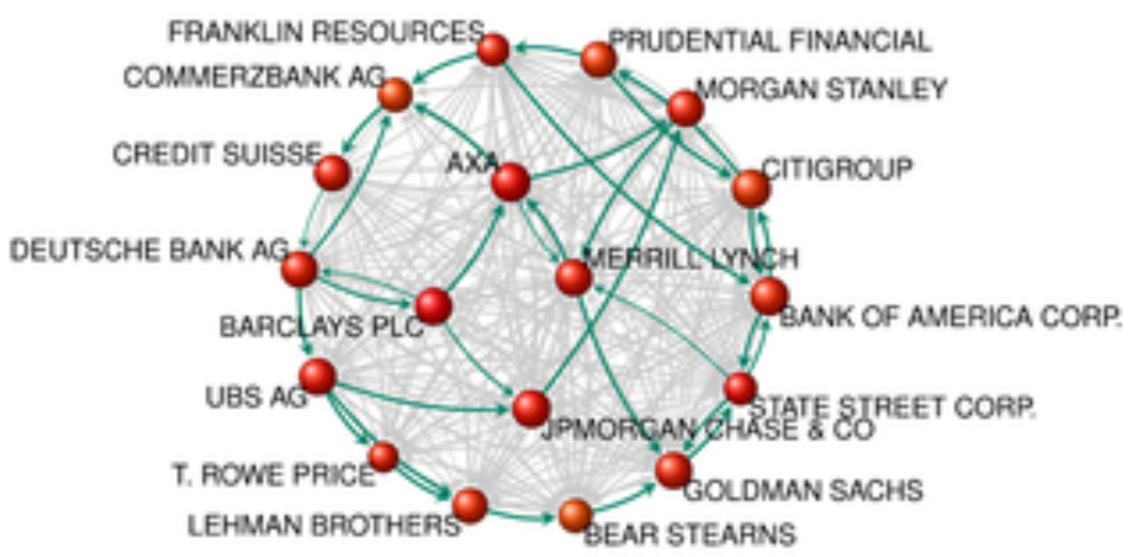

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Inequality and Stability

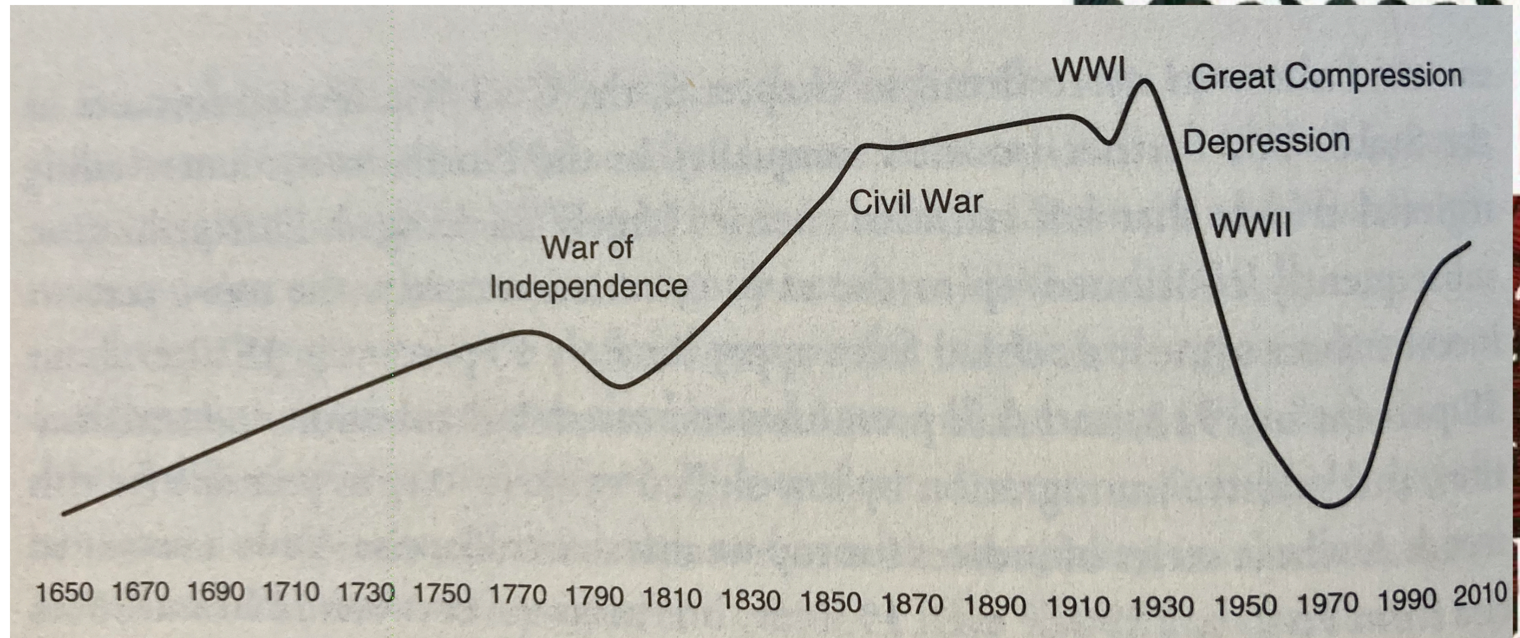
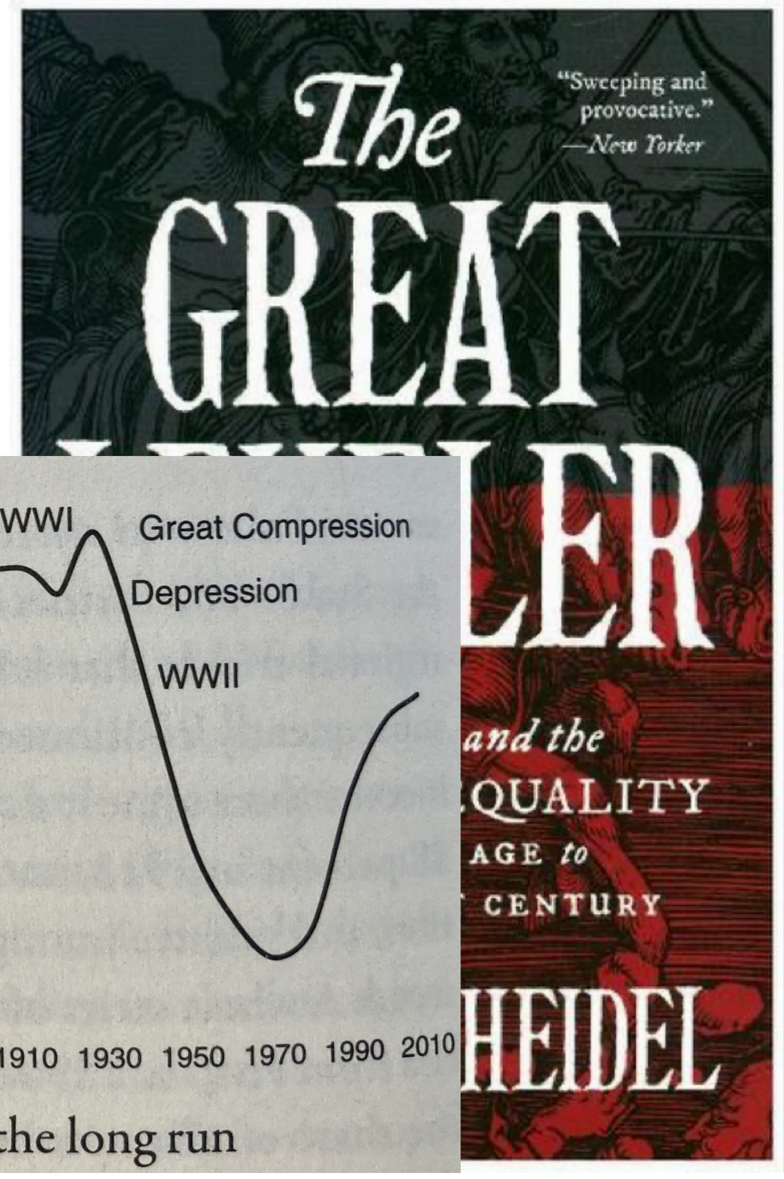


Figure 3.5 Inequality trends in the United States in the long run



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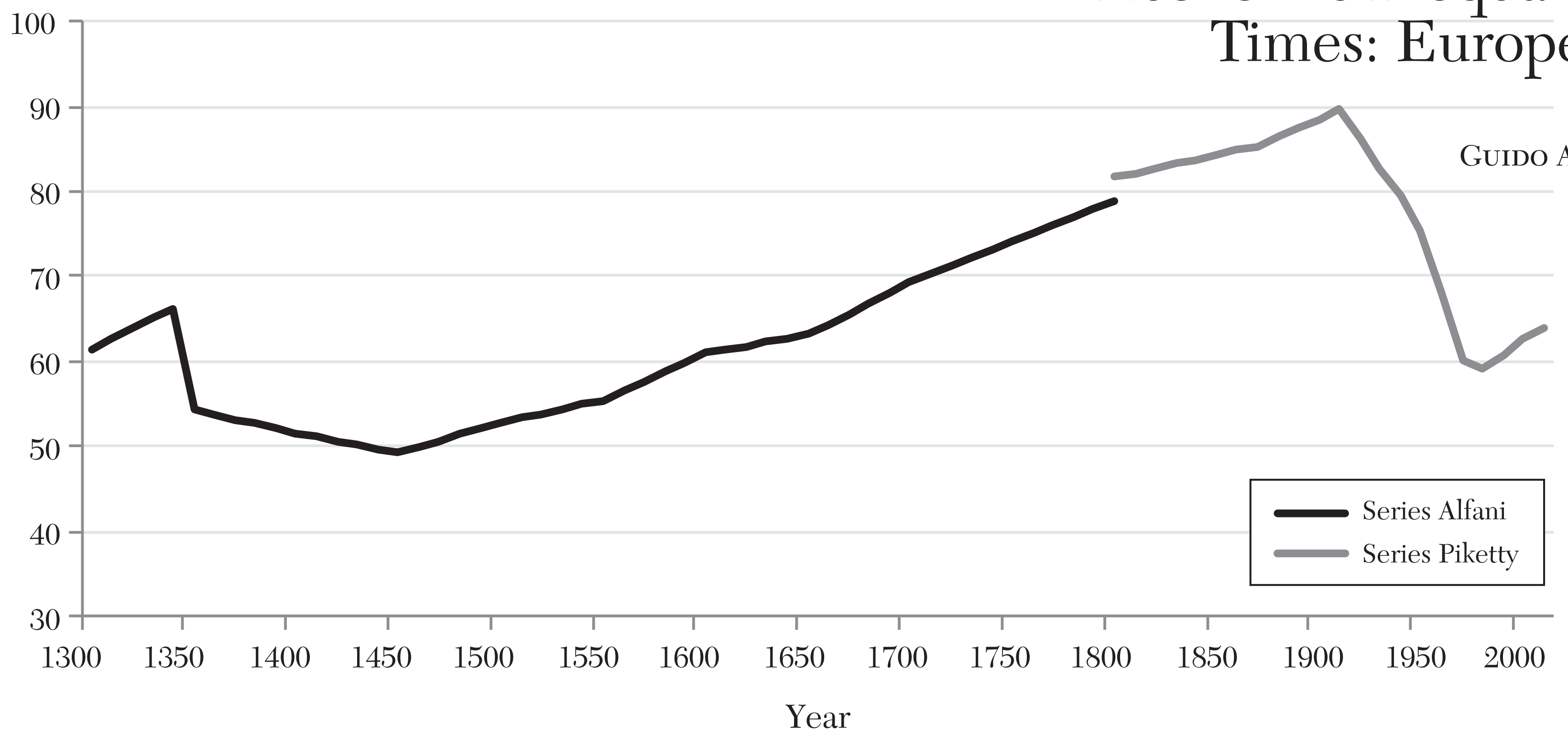

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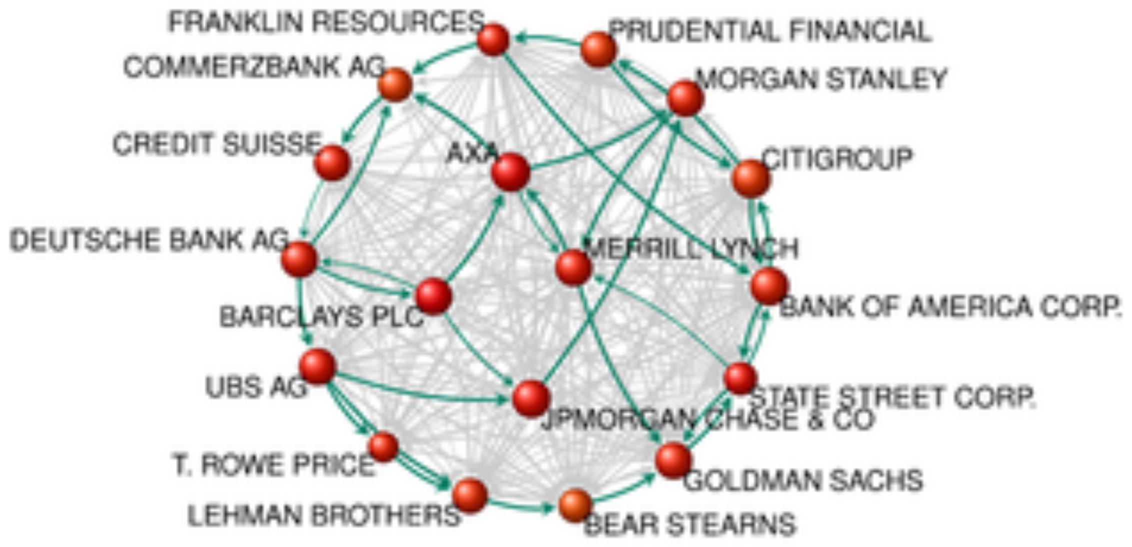

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Persistence: Florence 1427 vs. 2011

Table 2. Persistence in families' socioeconomic status

Surname	Euros (2011)	Occupation (1427)	% earnings (1427)	% wealth (1427)
5 richest in 2011:				
A	149,547	Member of shoemakers' guild	90%	89%
B	99,254	Member of silk guild (merchant or weaver)	97%	97%
C	95,881	Member of wool guild (manufacturer or merchant)	69%	65%
D	85,862	Messer (lawyer)	94%	93%
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https://www.vox.com/2016/5/18/11691818/barone-mocetti-florence

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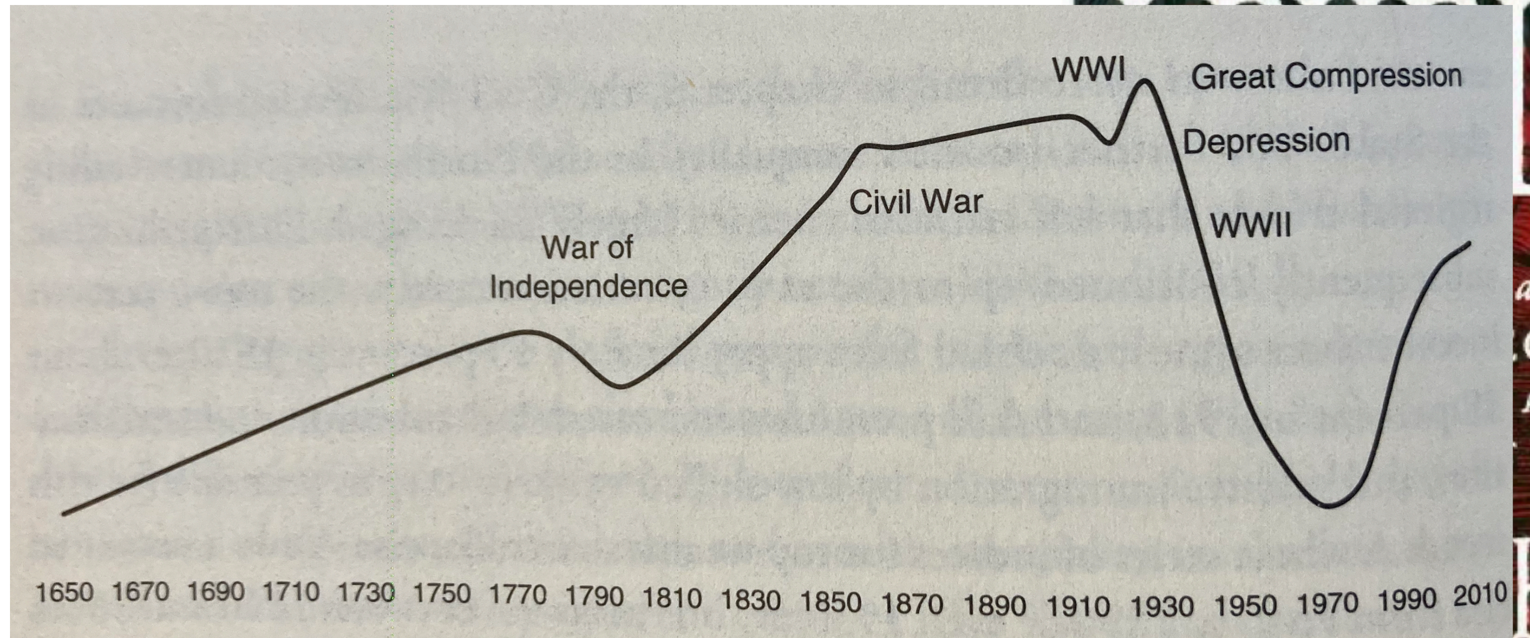

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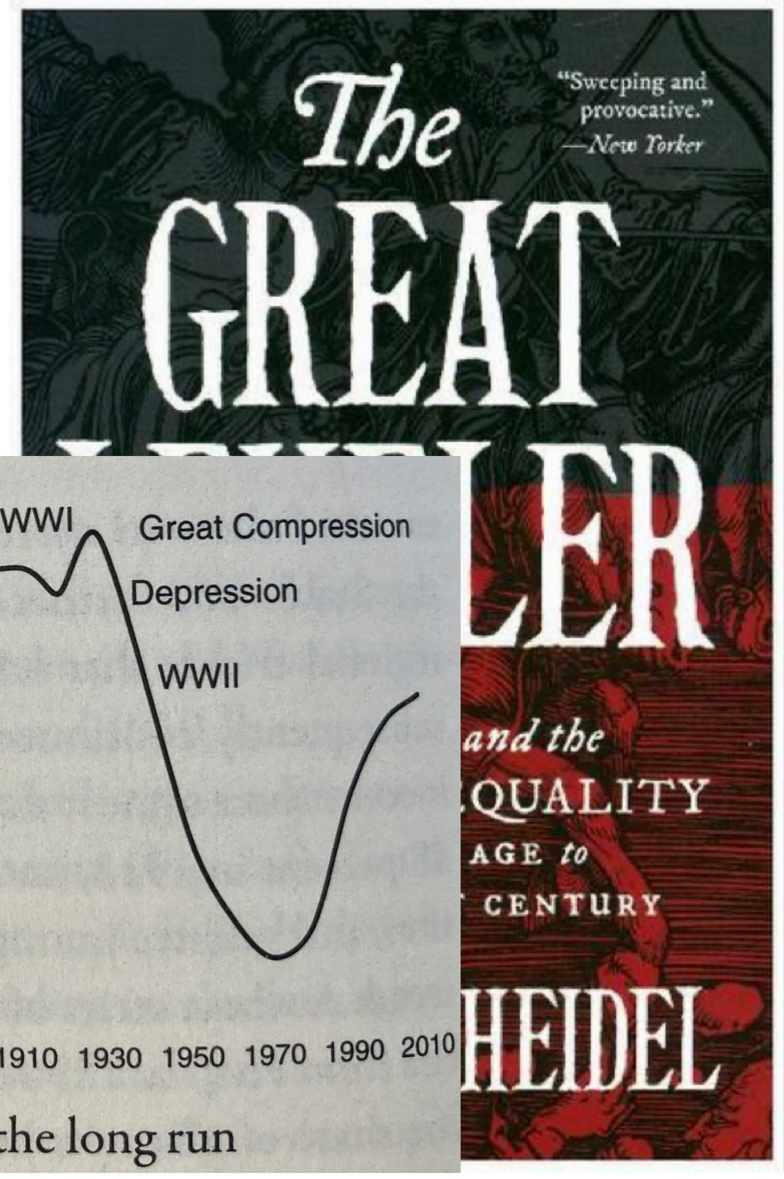

Cumulative advantage: some empirical traces

Persistent poverty effects:
Another consequence of positive feedback

WHY DO PEOPLE STAY POOR?*

CLARE BALBONI
ORIANA BANDIERA
ROBIN BURGESS
MAITREESH GHATAK
ANTON HEIL

There are two broad views as to why people stay poor. One emphasizes differences in fundamentals, such as ability, talent, or motivation. The poverty traps view emphasizes differences in opportunities that stem from access to wealth. To test these views, we exploit a large-scale, randomized asset transfer and an 11-year panel of 6,000 households who begin in extreme poverty. The setting is rural Bangladesh, and the assets are cows. The data support the poverty traps view—we identify a threshold level of initial assets above which households accumulate assets, take on better occupations (from casual labor in agriculture or domestic services to running small livestock businesses), and grow out of poverty. The reverse happens for those below the threshold. Structural estimation of an occupational choice model reveals that almost all beneficiaries are misallocated in the work they do at baseline and that the gains arising from eliminating misallocation would far exceed the program costs. Our findings imply that large transfers, which create better jobs for the poor, are an effective means of getting people out of poverty traps and reducing global poverty. *JEL Codes:* I32, J22, J24, O12.

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Ongoing concentration

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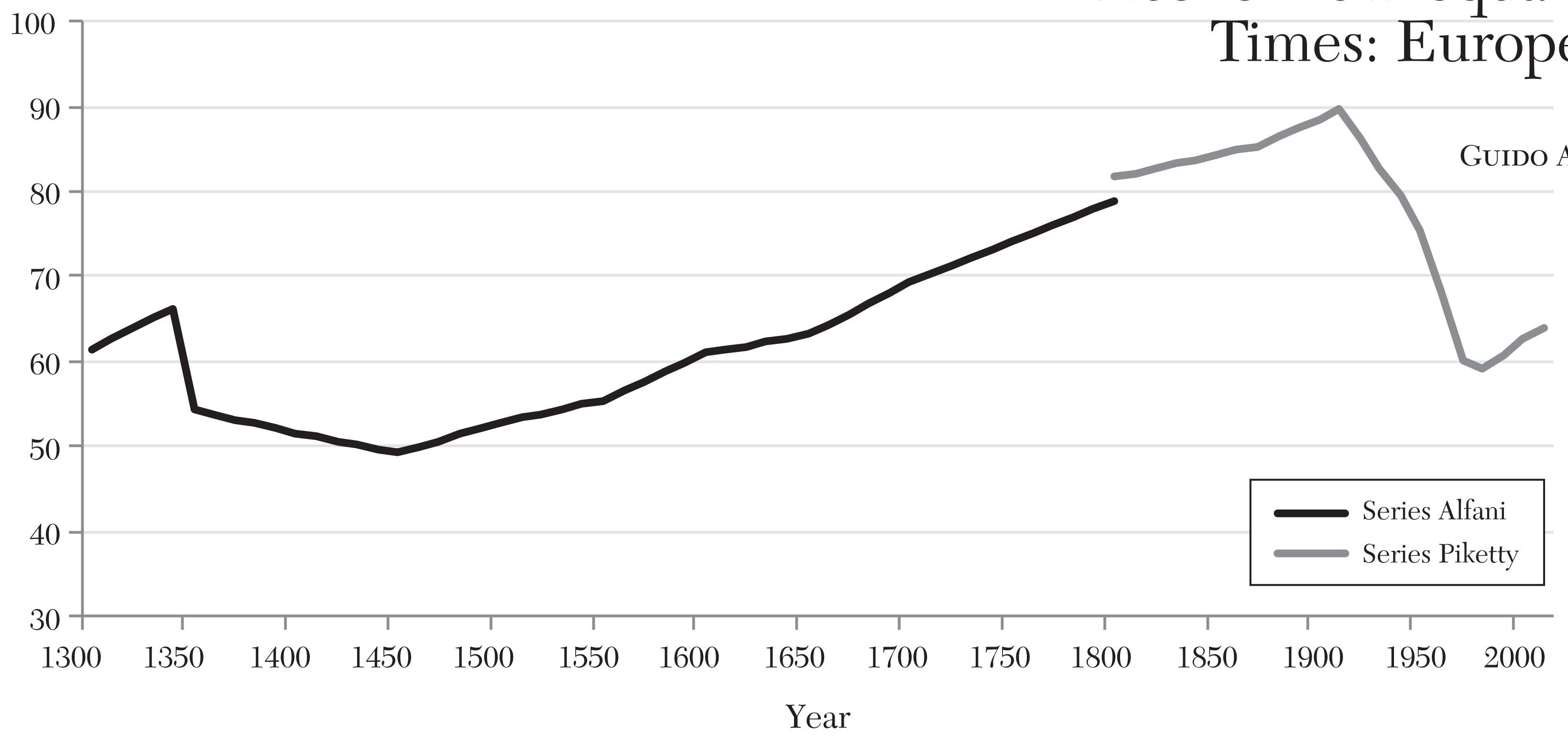

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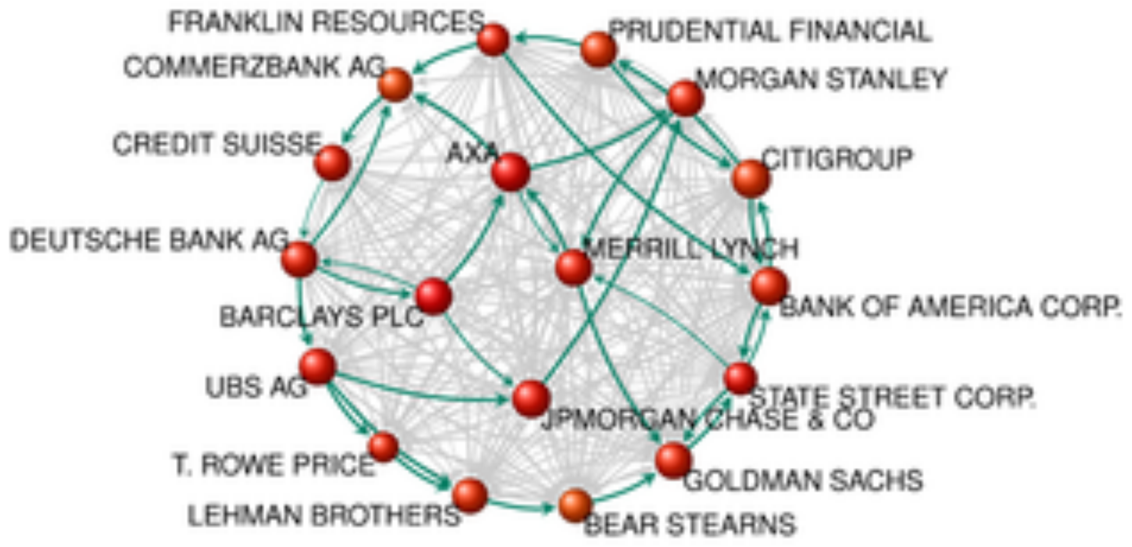

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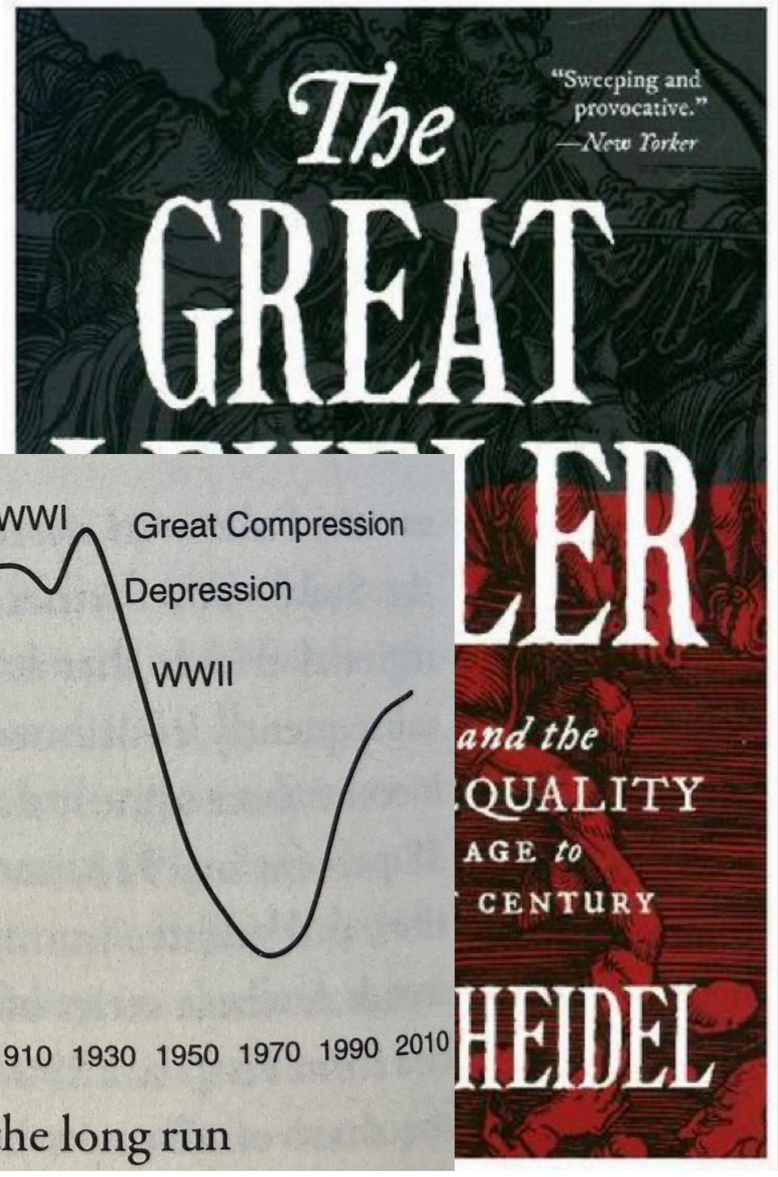

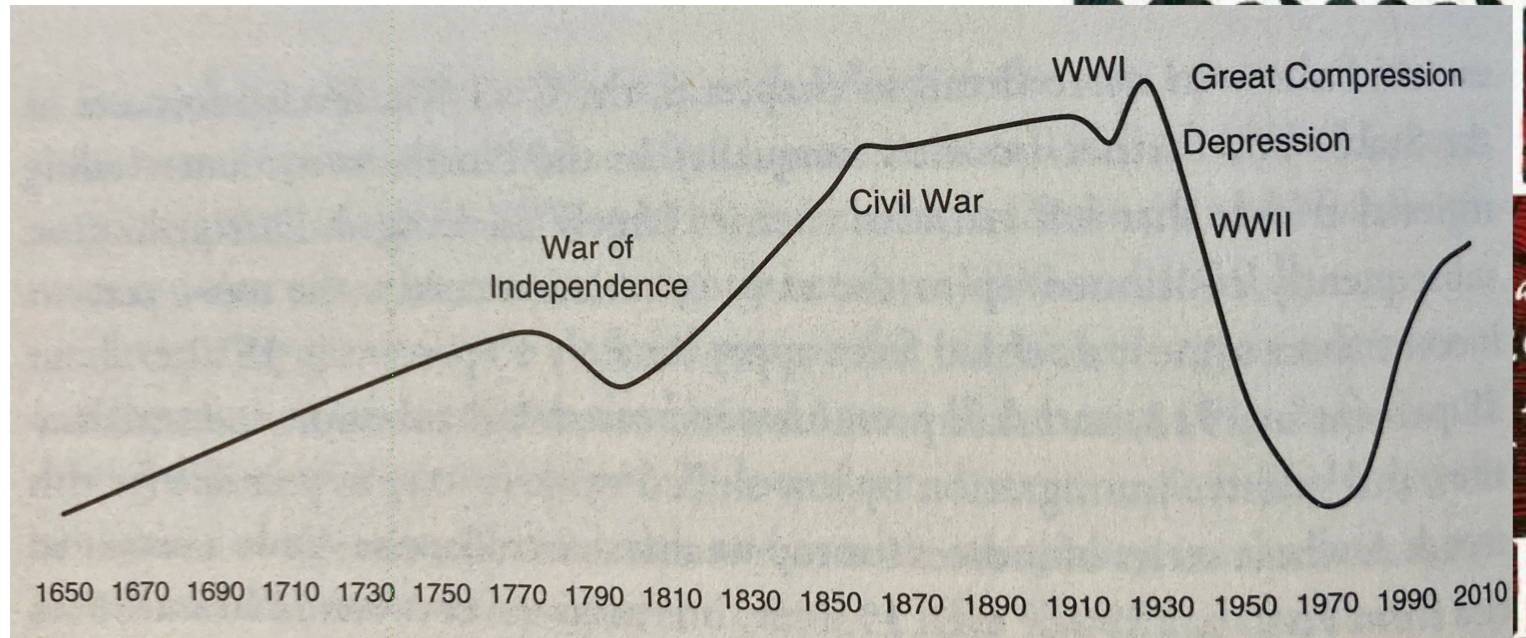

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ifso institute for socio-economics
UNIVERSITÄT DUISBURG ESSEN
Open-Minded

Random multiplicative dynamics
(aka „Gibrat models")

A simple model of random multiplicative growth

- The classic „Gibrat model" looks like…

$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} \qquad \text{with} \qquad r_{i,t} \sim \mathcal{N}\left(\mu, \sigma_G\right)$$

  - Simpler formulation, same **explosive** properties…
  - Starting from **perfect equality**: $w_{i,0} = 10 \; \forall \; i$
  - A main difference is that $0.05 = \sigma_G \gg \sigma = 0.001$ – **random walk** more pronounced.
  - Similar overall pattern, different dynamics.

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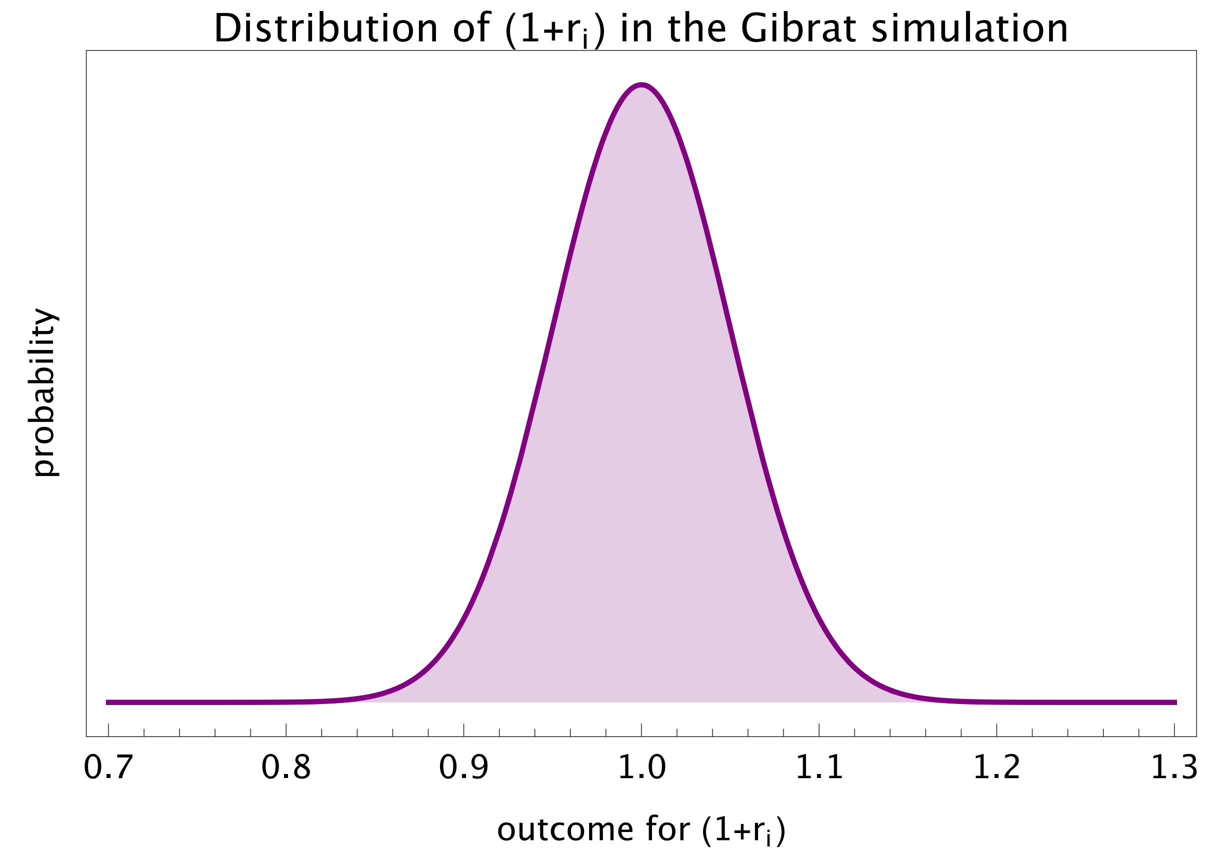

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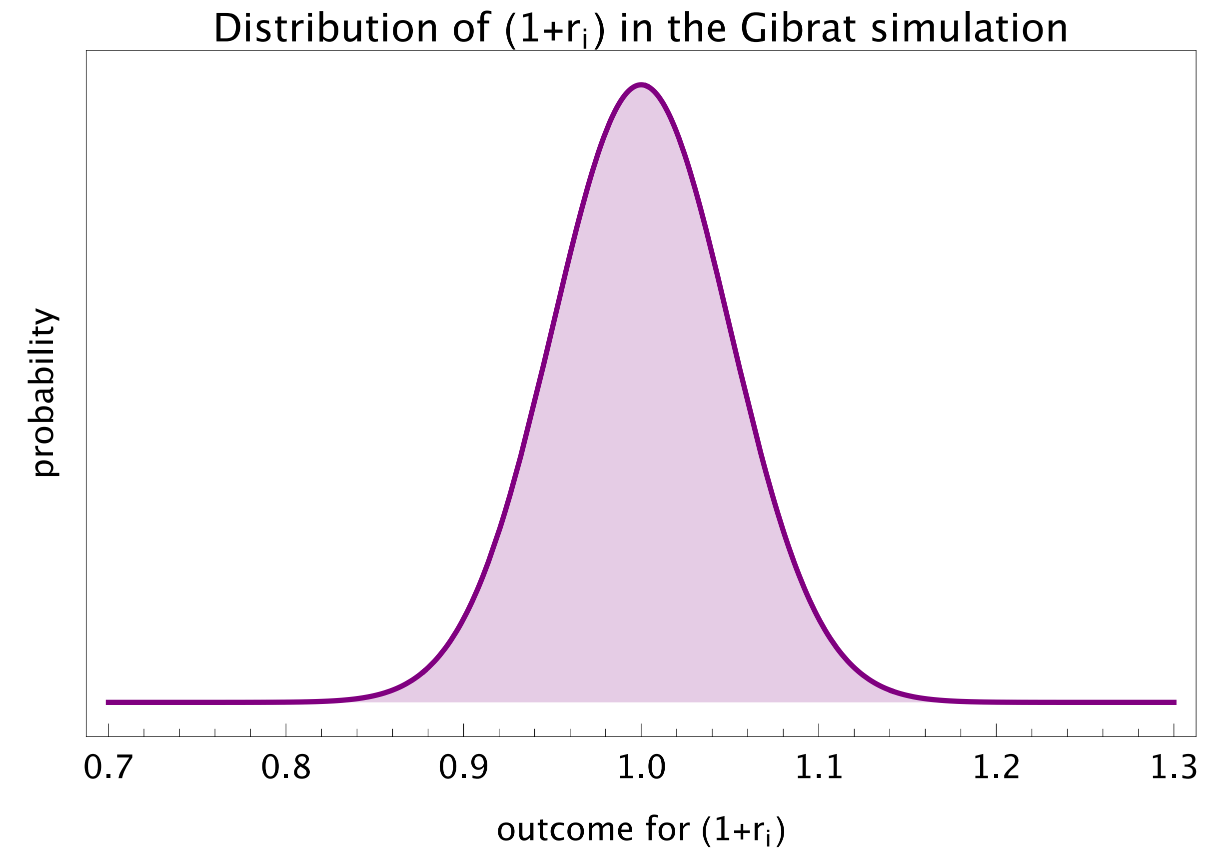

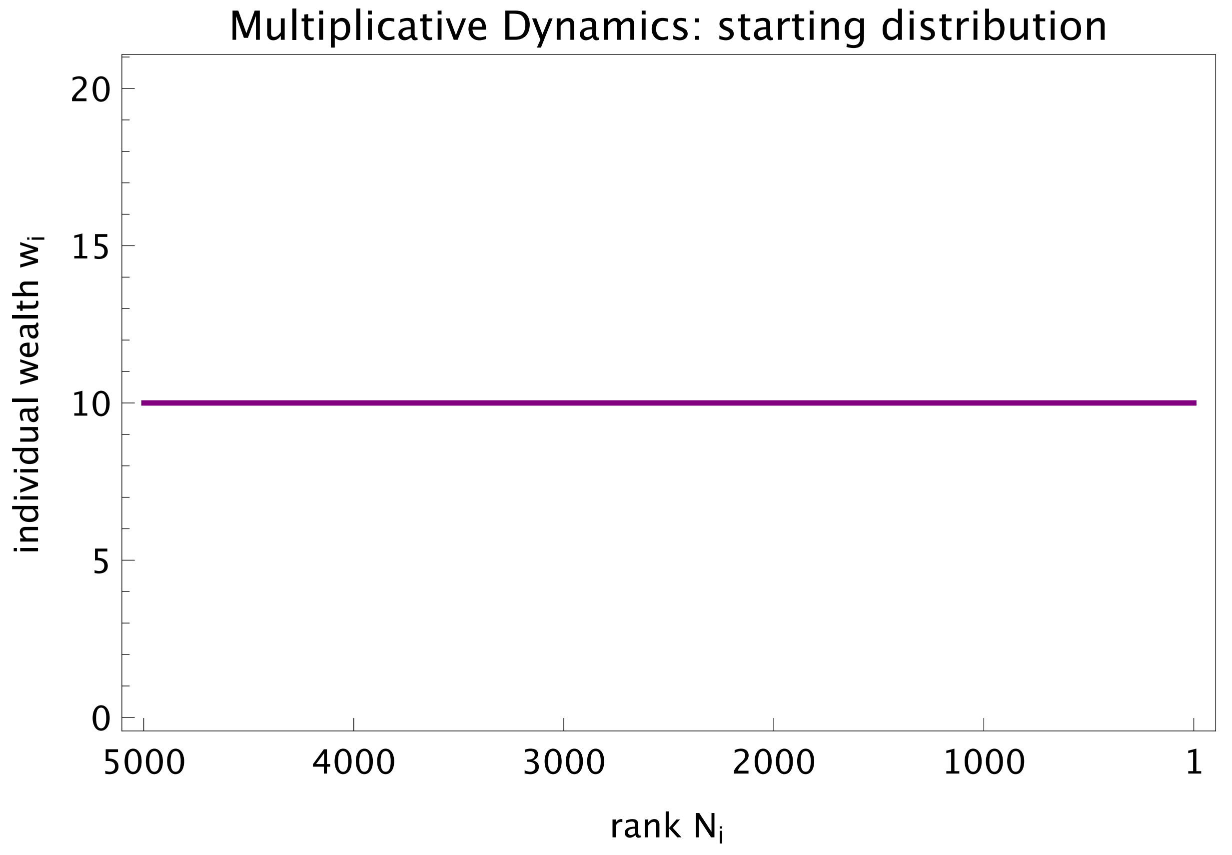

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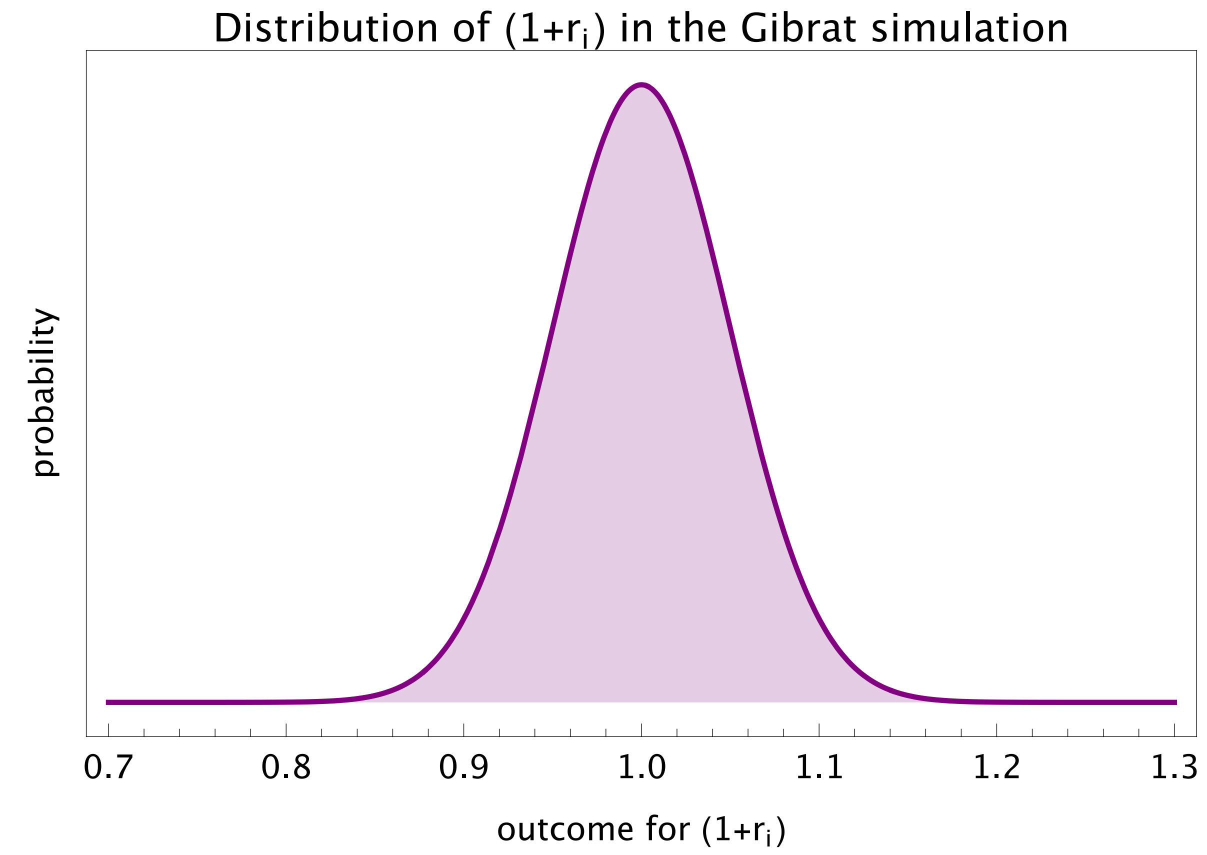

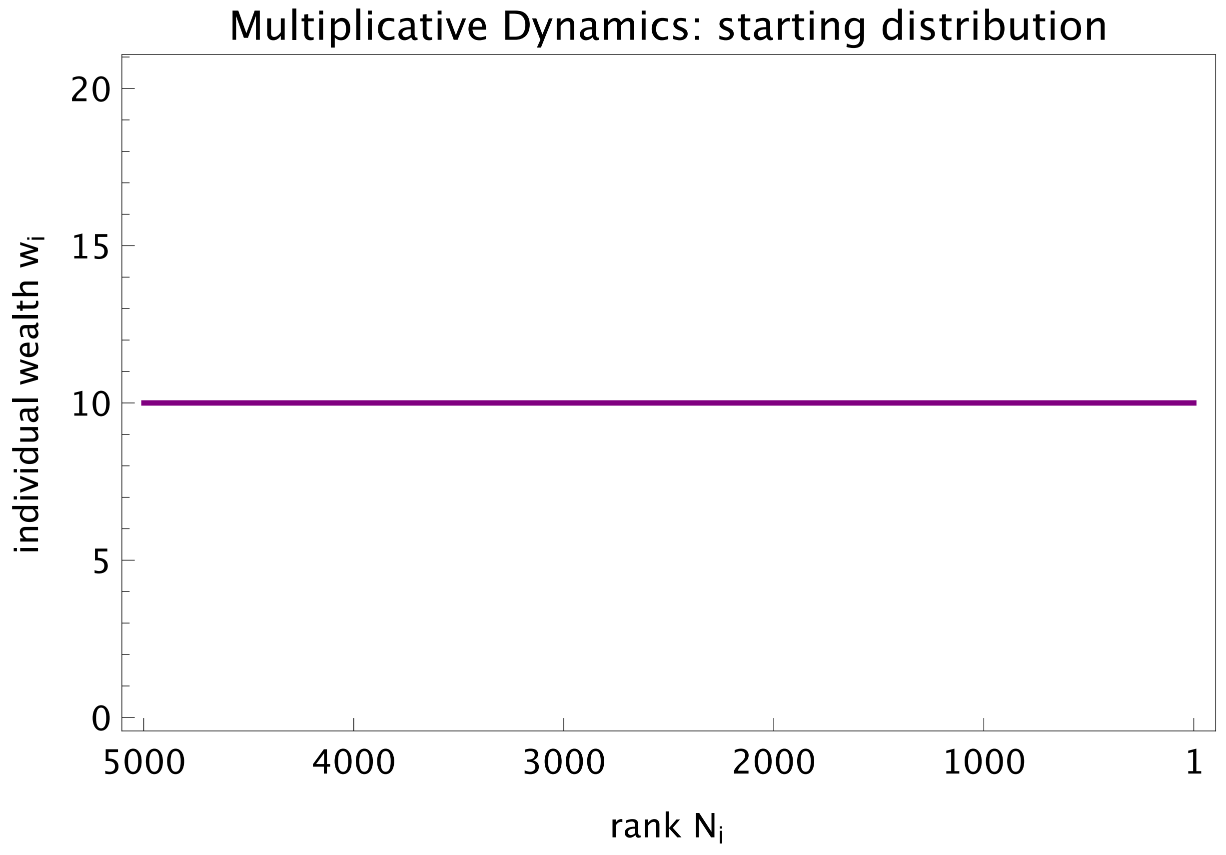

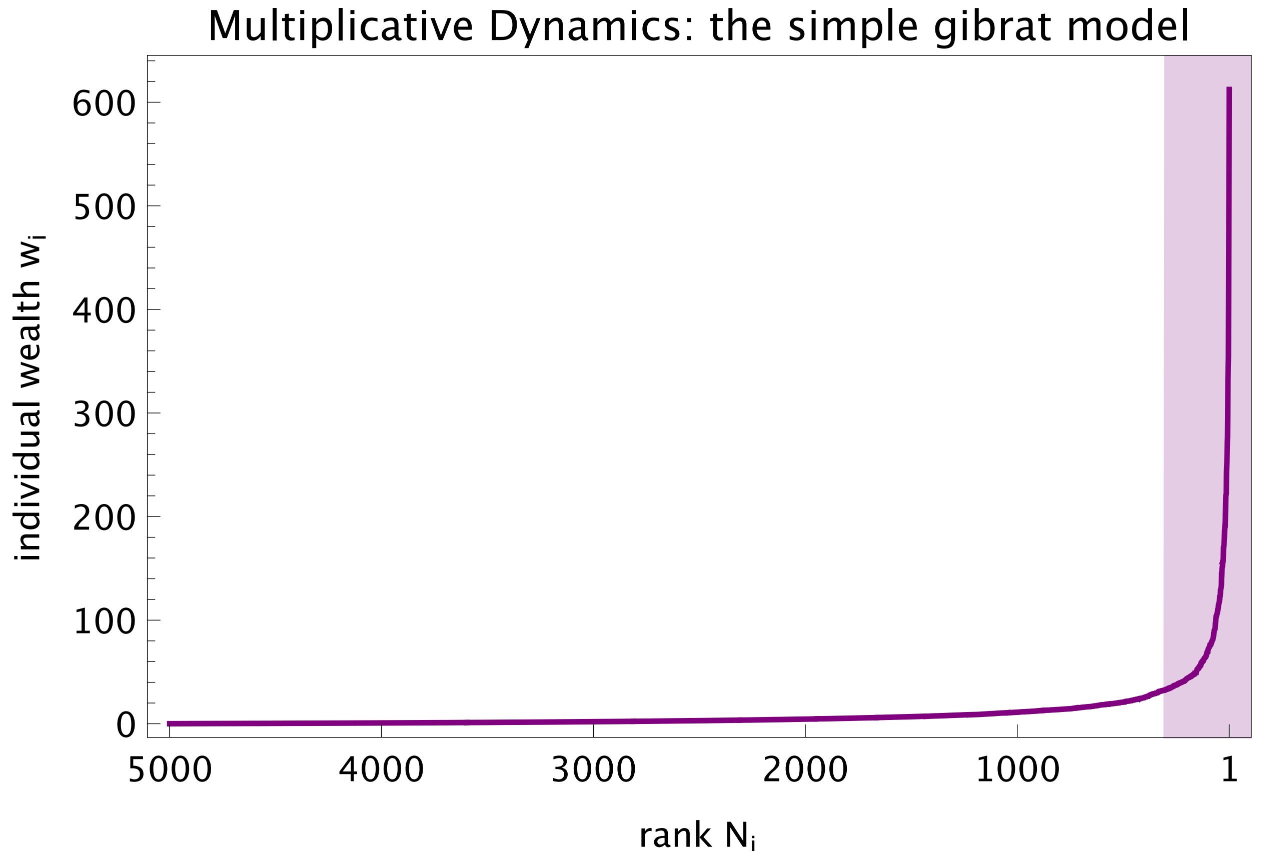

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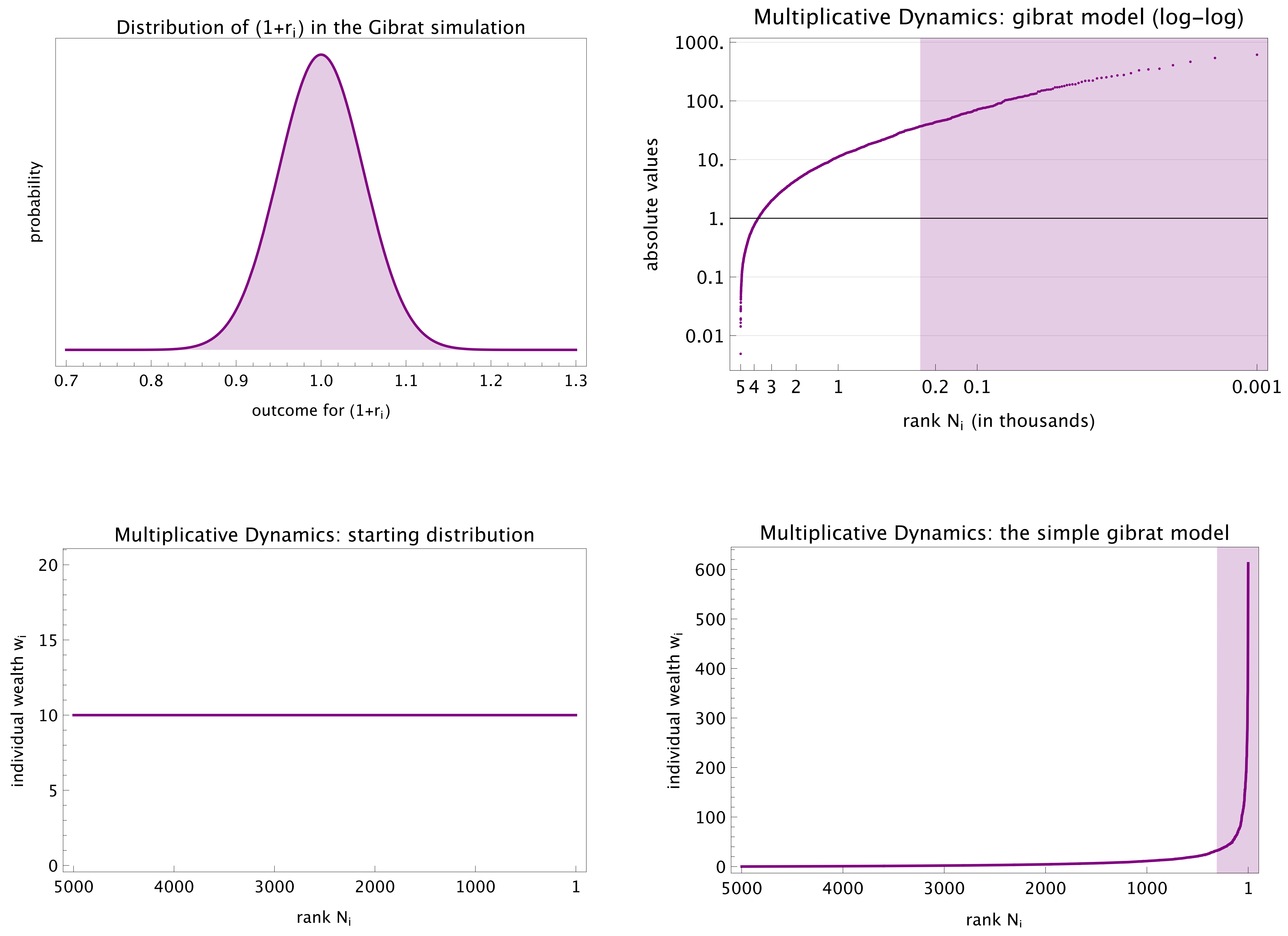
$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} \qquad \text{with} \qquad r_{i,t} \sim \mathcal{N}\left(\mu, \sigma_G\right)$$

  - Simpler formulation, same **explosive** properties…
  - Starting from **perfect equality**: $w_{i,0} = 10 \ \forall \ i$
  - A main difference is that $0.05 = \sigma_G \gg \sigma = 0.001$ – **random walk** more pronounced.
  - Similar overall pattern, different dynamics.



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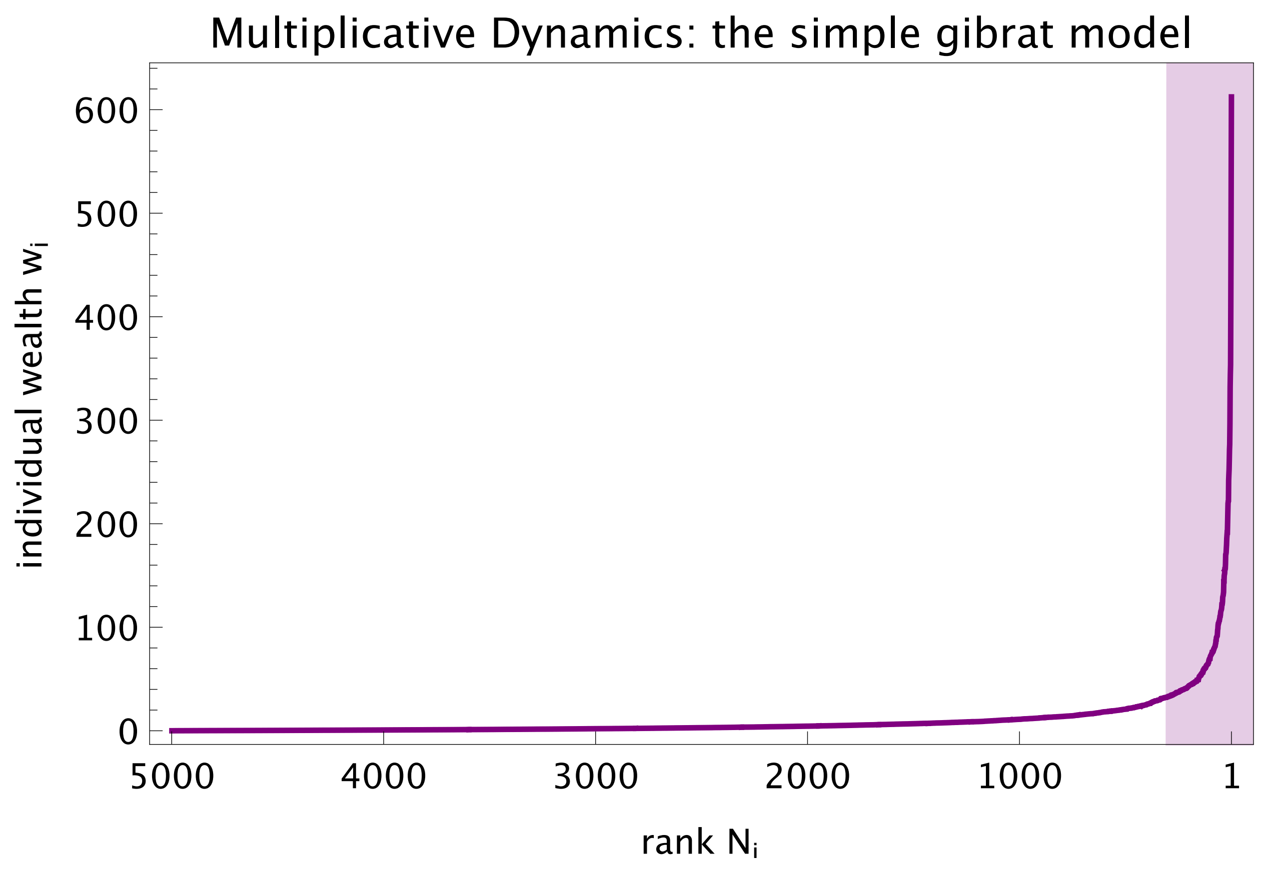

- Path Dependency?
  - In cumulative advantage models: hard path-dependencies (only one ‚eternal' change in direction) ➝ non-ergodic dynamics (averages between elements do not inform about trajectories over time)
  - Here we have a multiplicative random walk: in eternity, everybody will rise & fall.
  - But think about time-scales! If a participant is born in round 800 and lives for 60 periods, the world will be super-path-dependent ➝ quasi-non-ergodic dynamics

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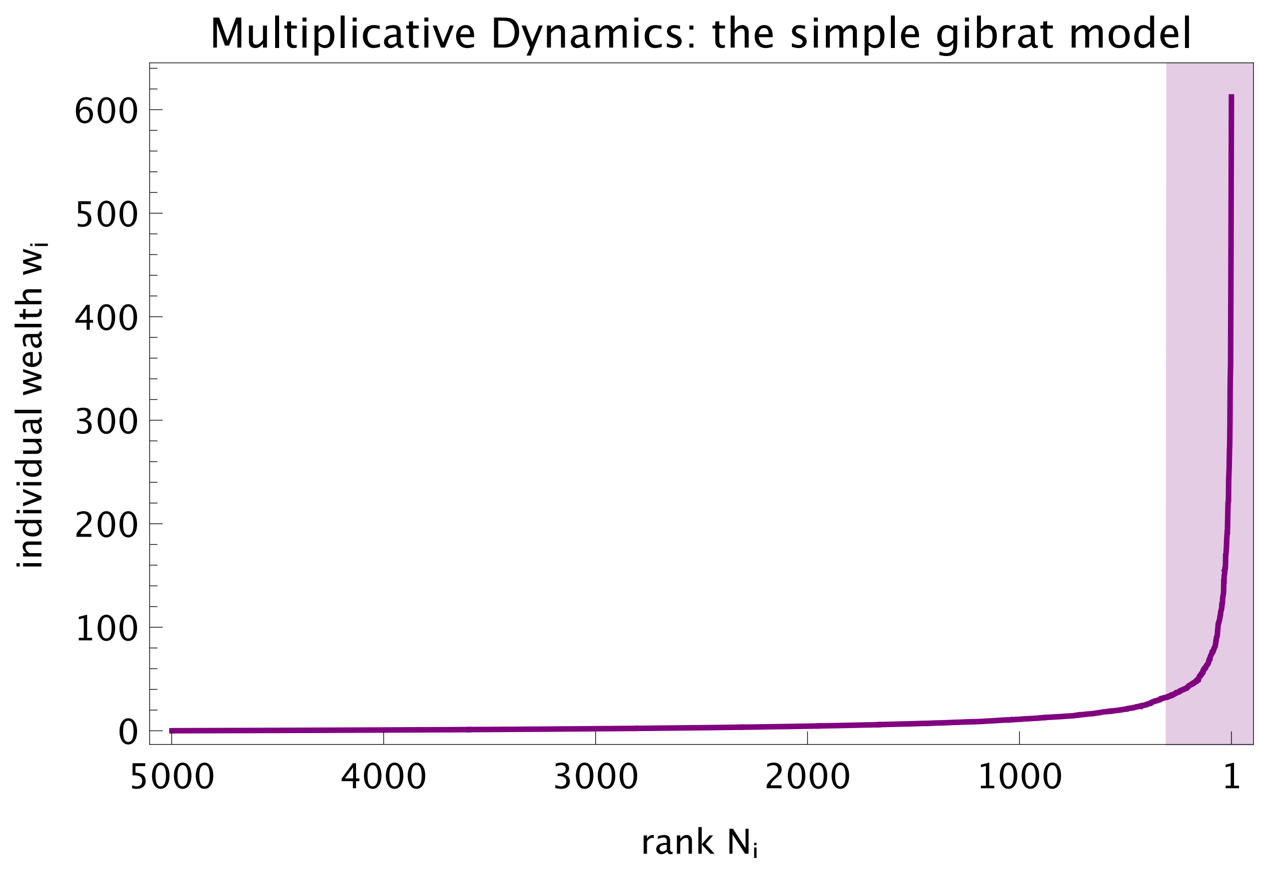

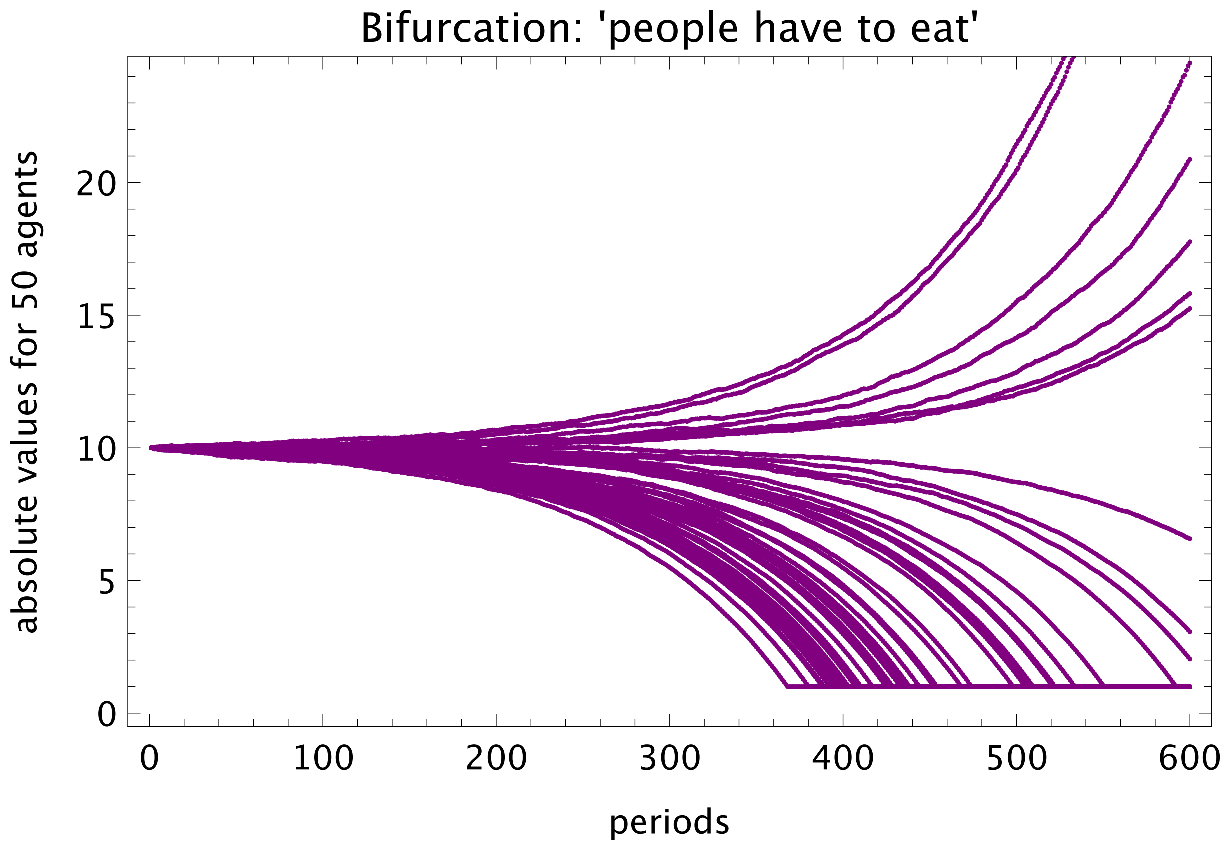

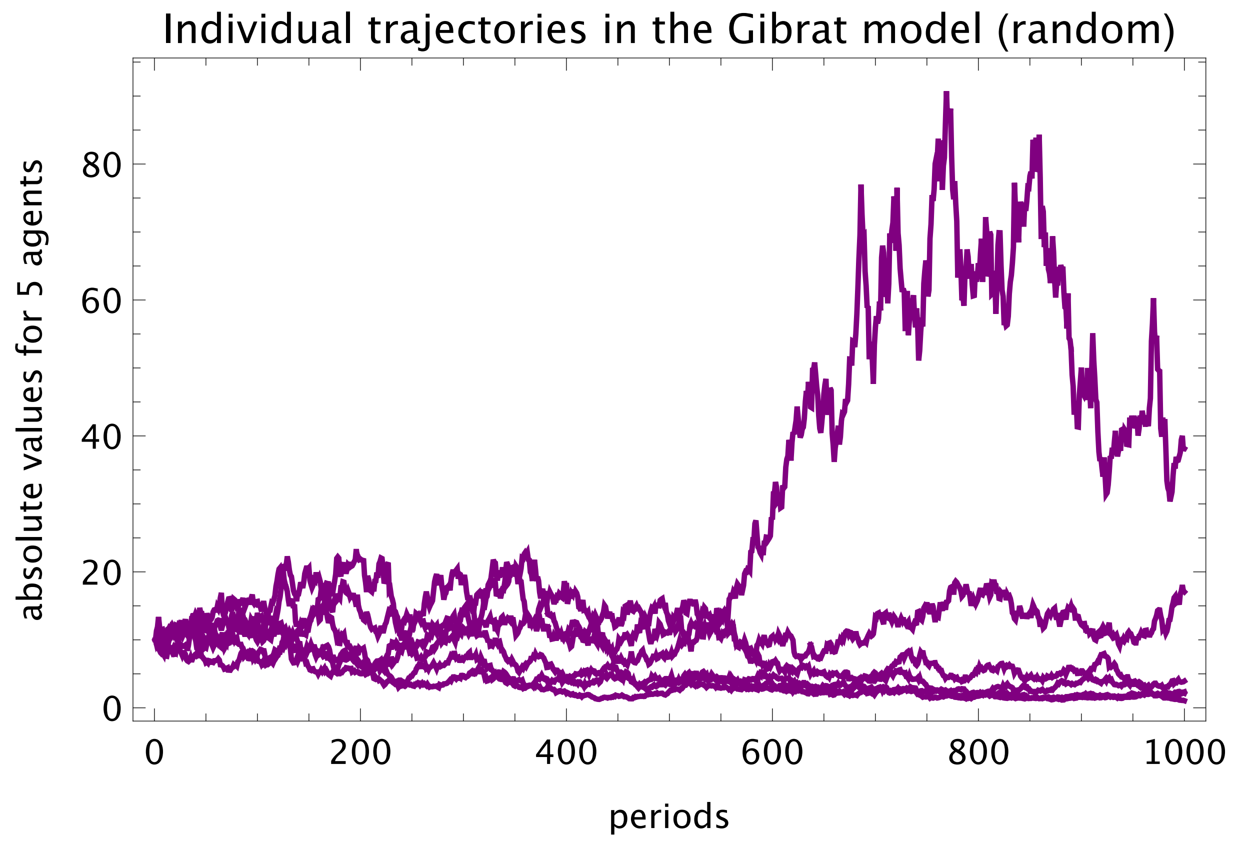

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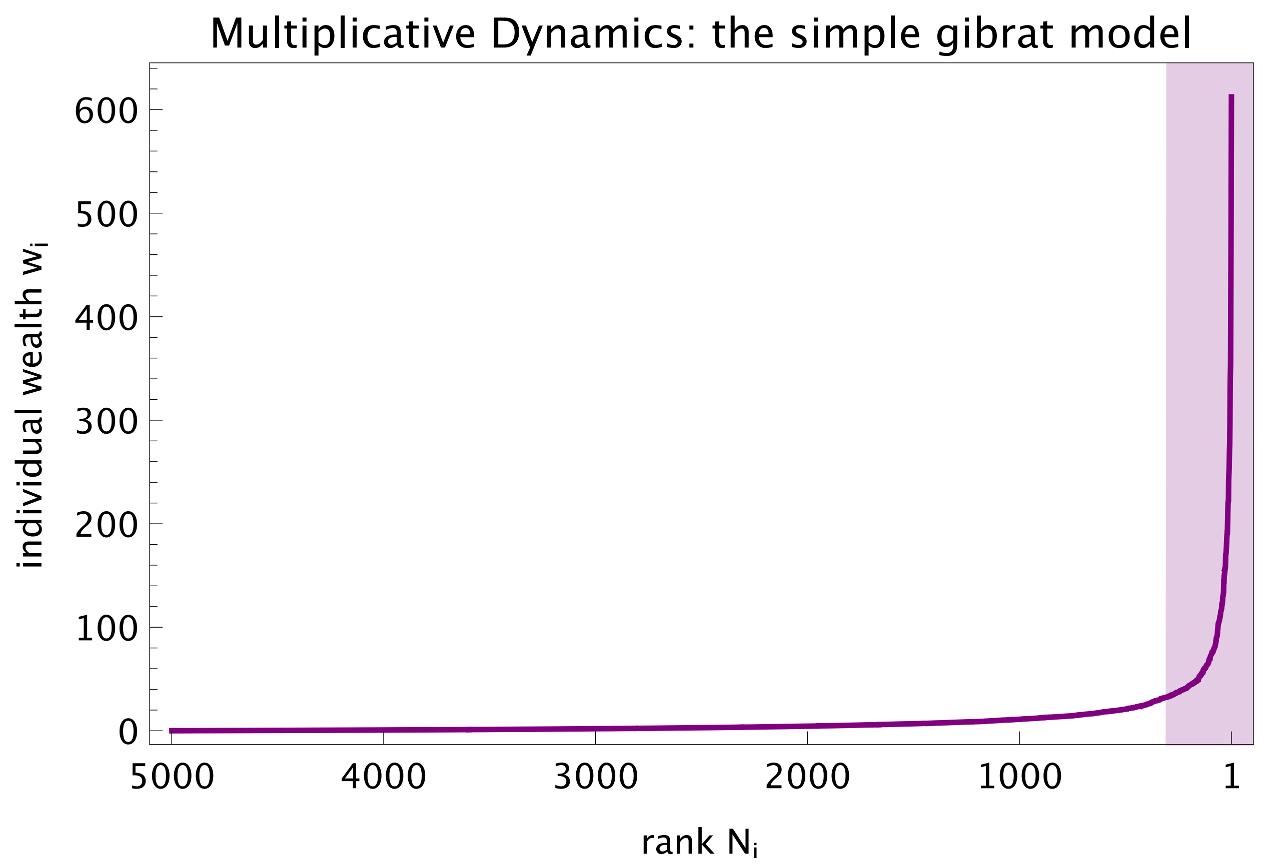

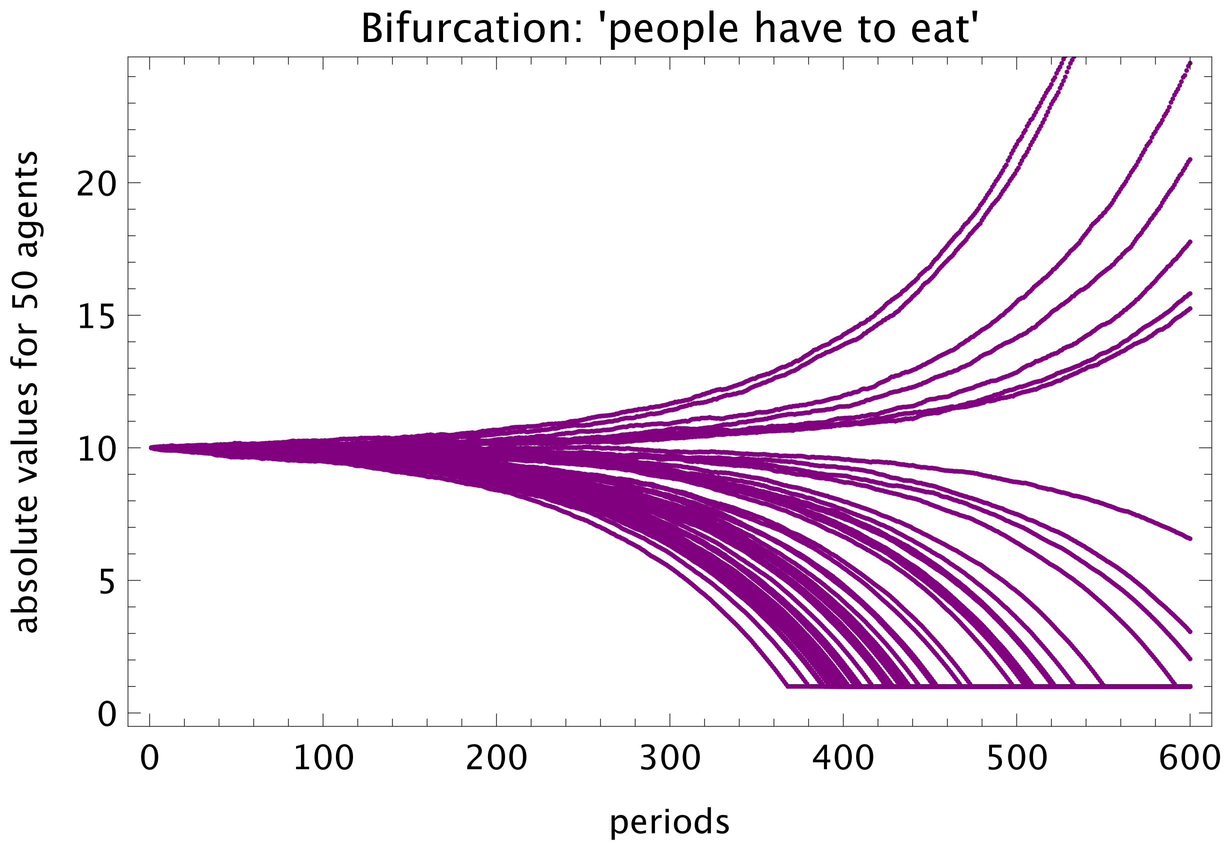

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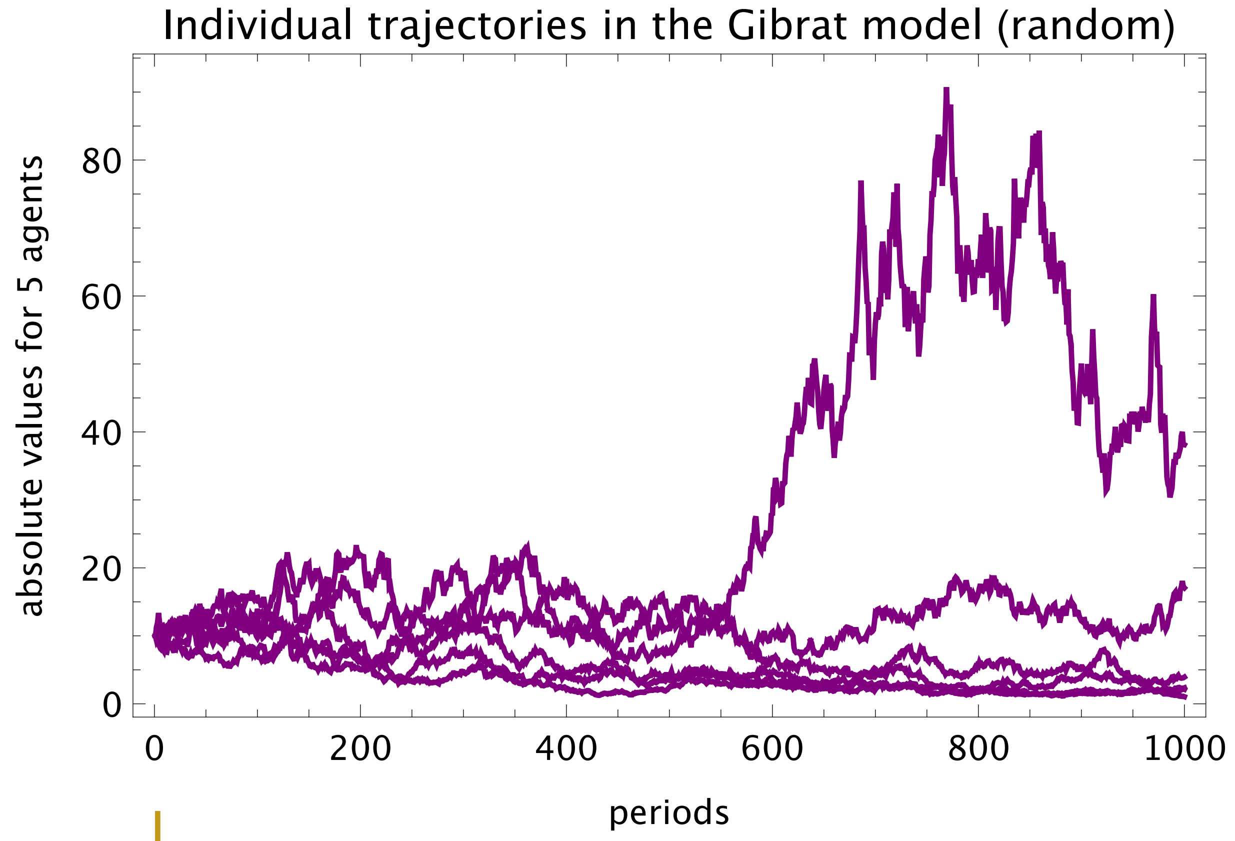

Calculating probabilities: If $w_j > w_i = 1$, then the probability of j staying richer is always $p > 1/2$.

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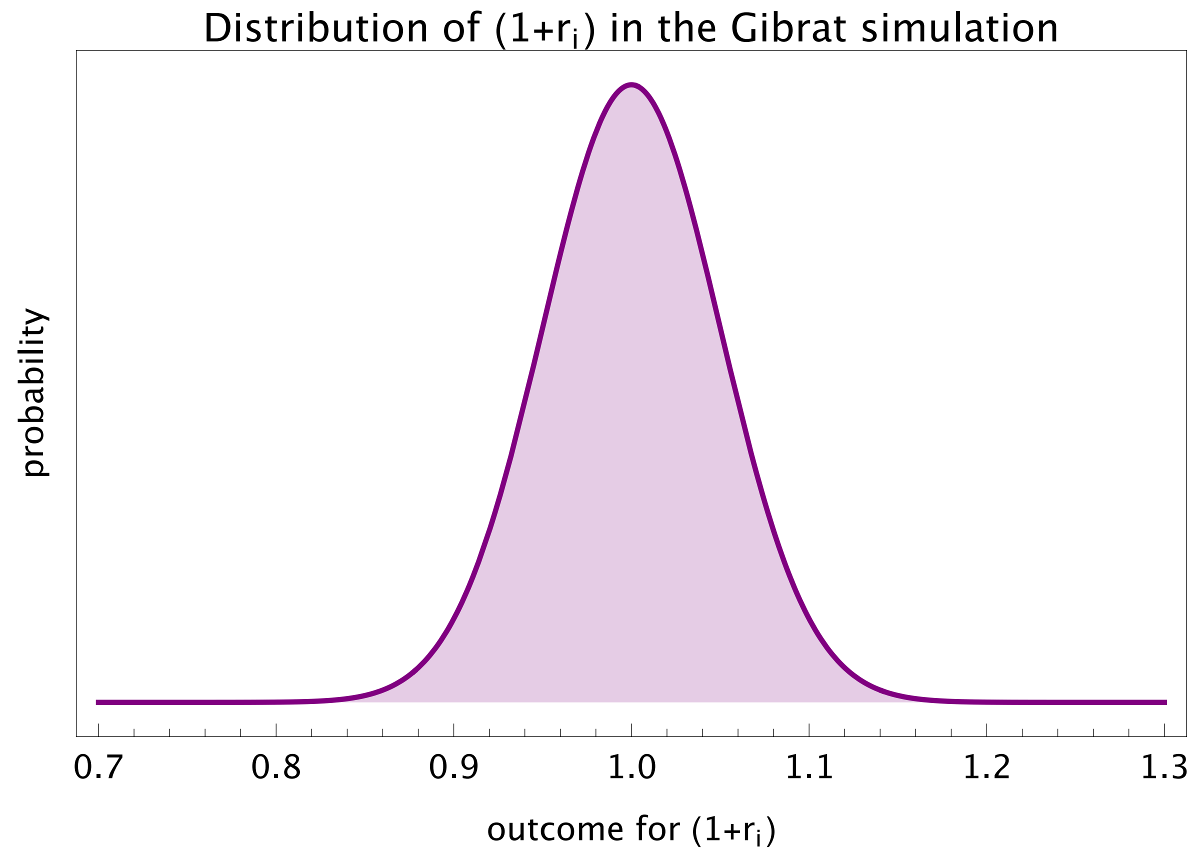

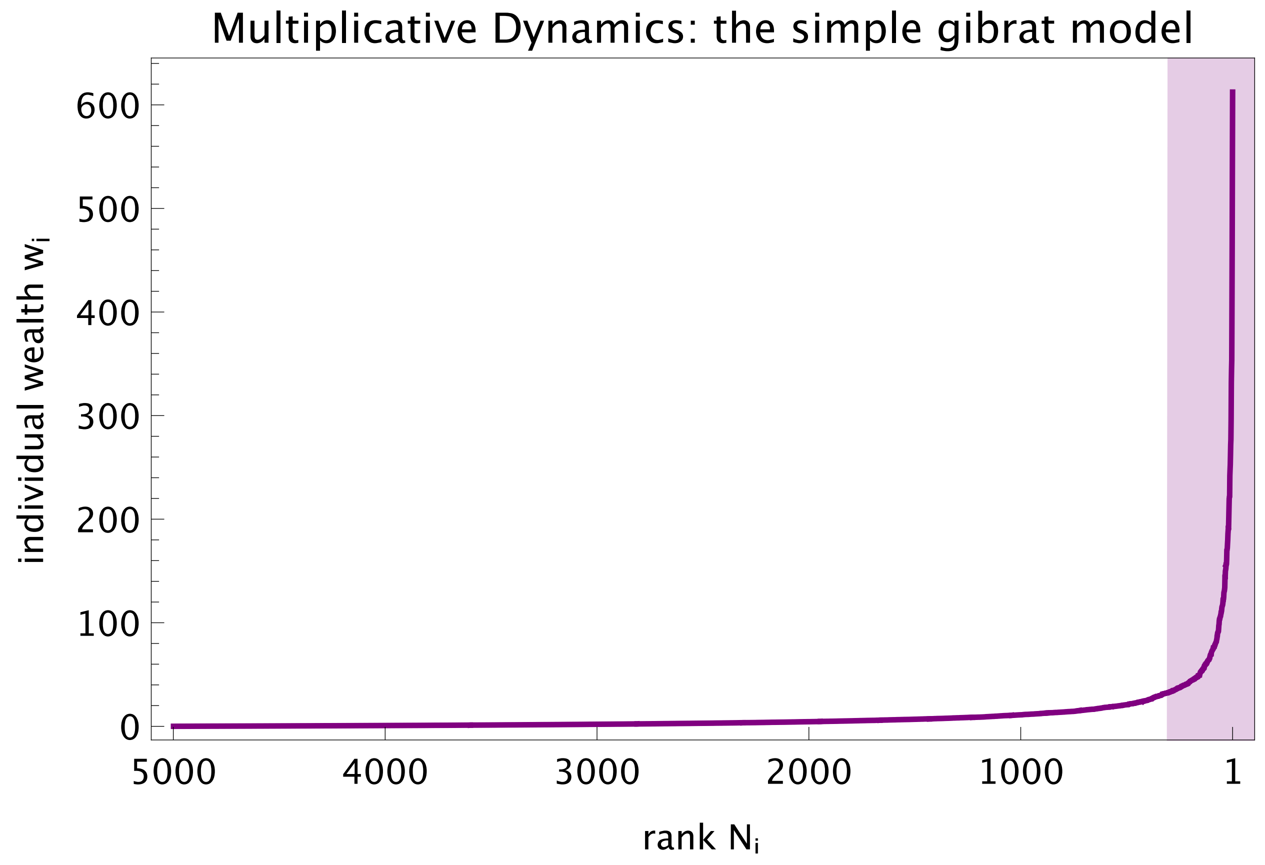

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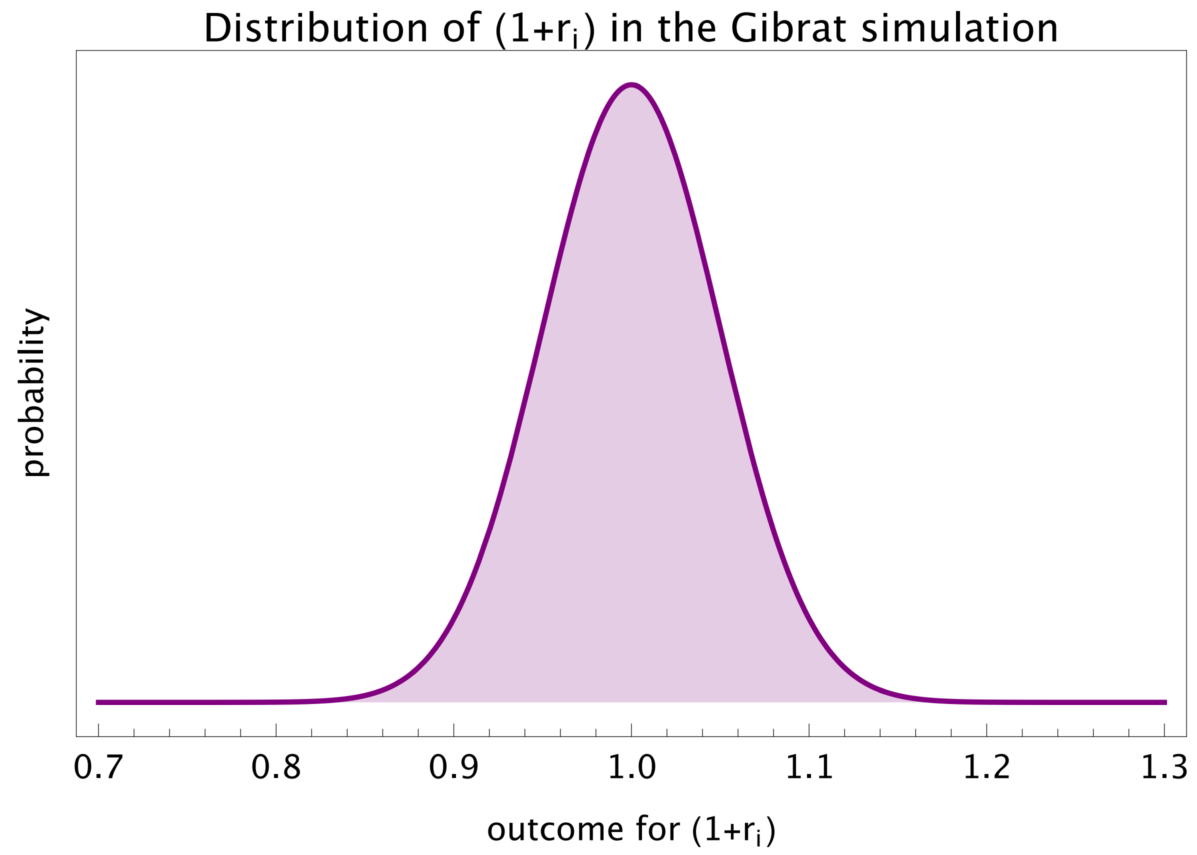

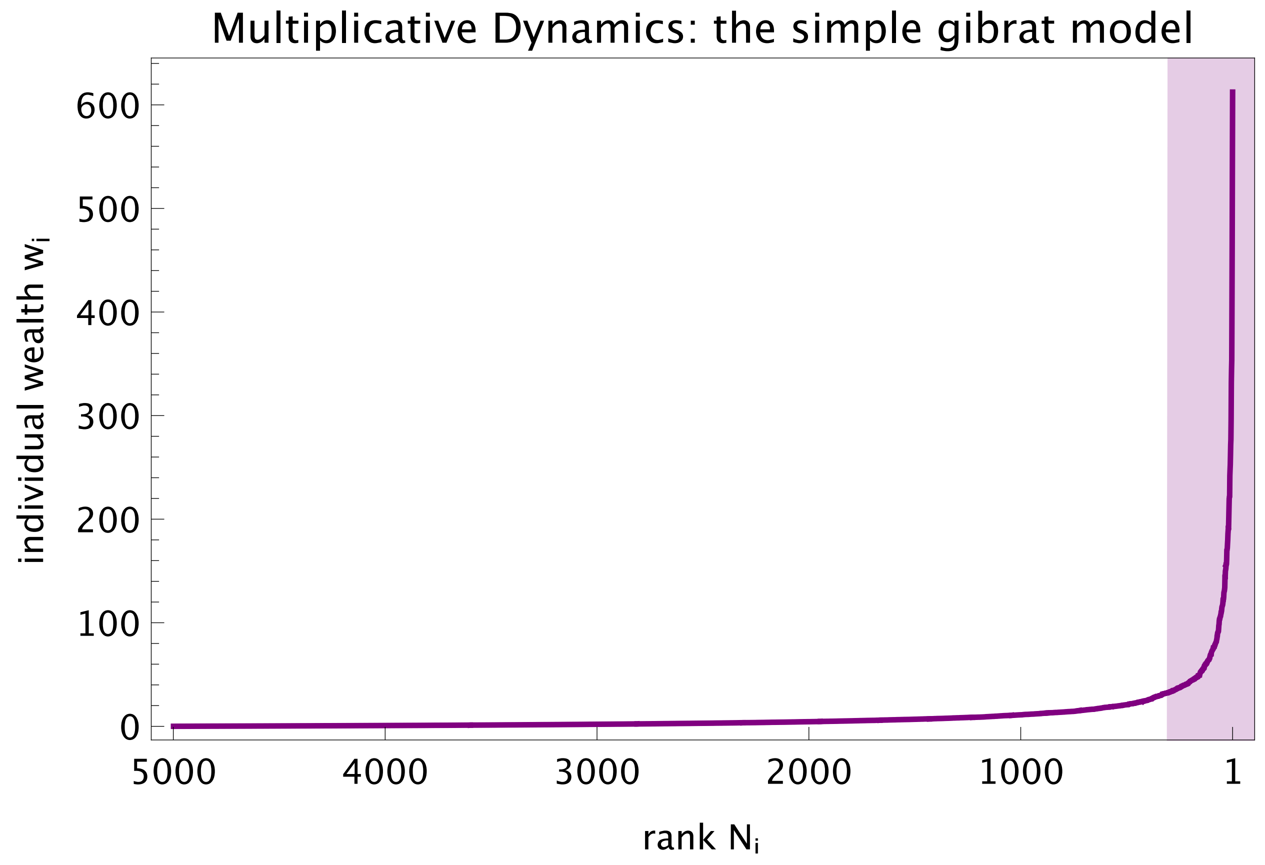

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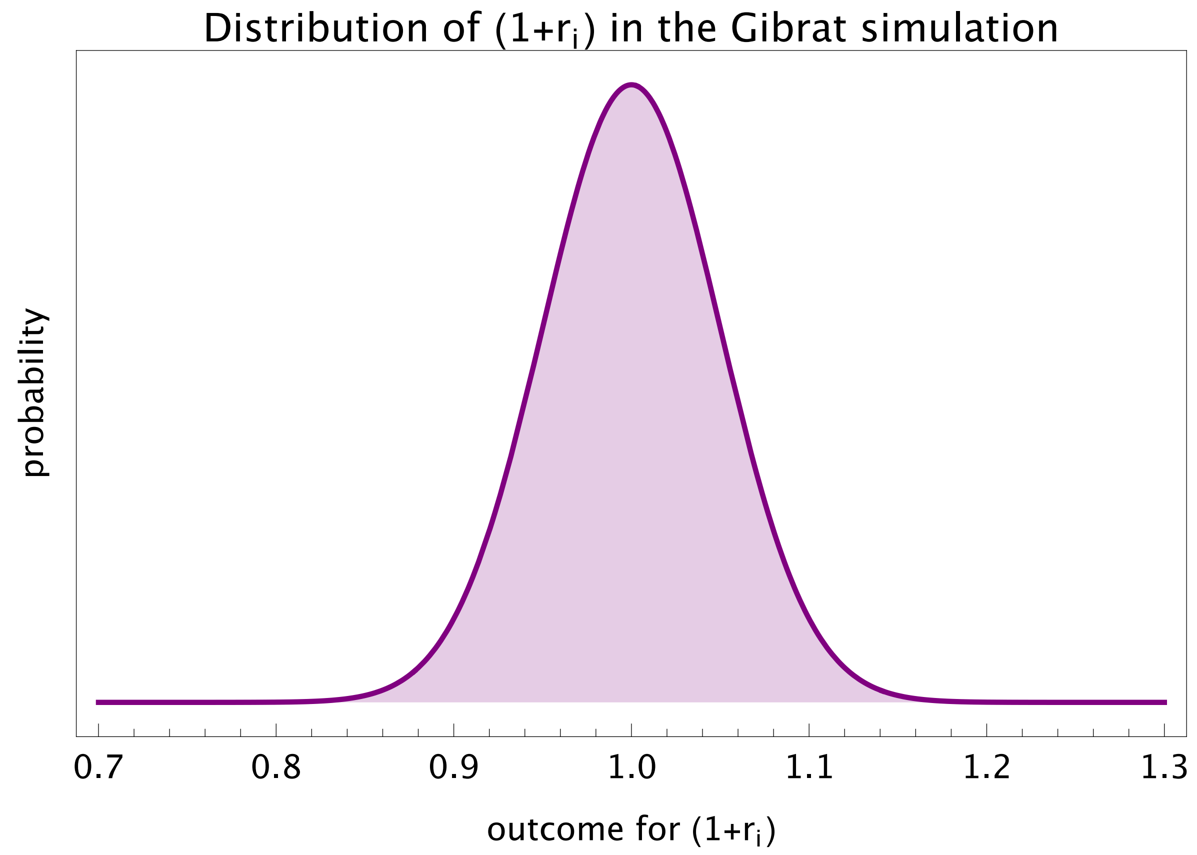

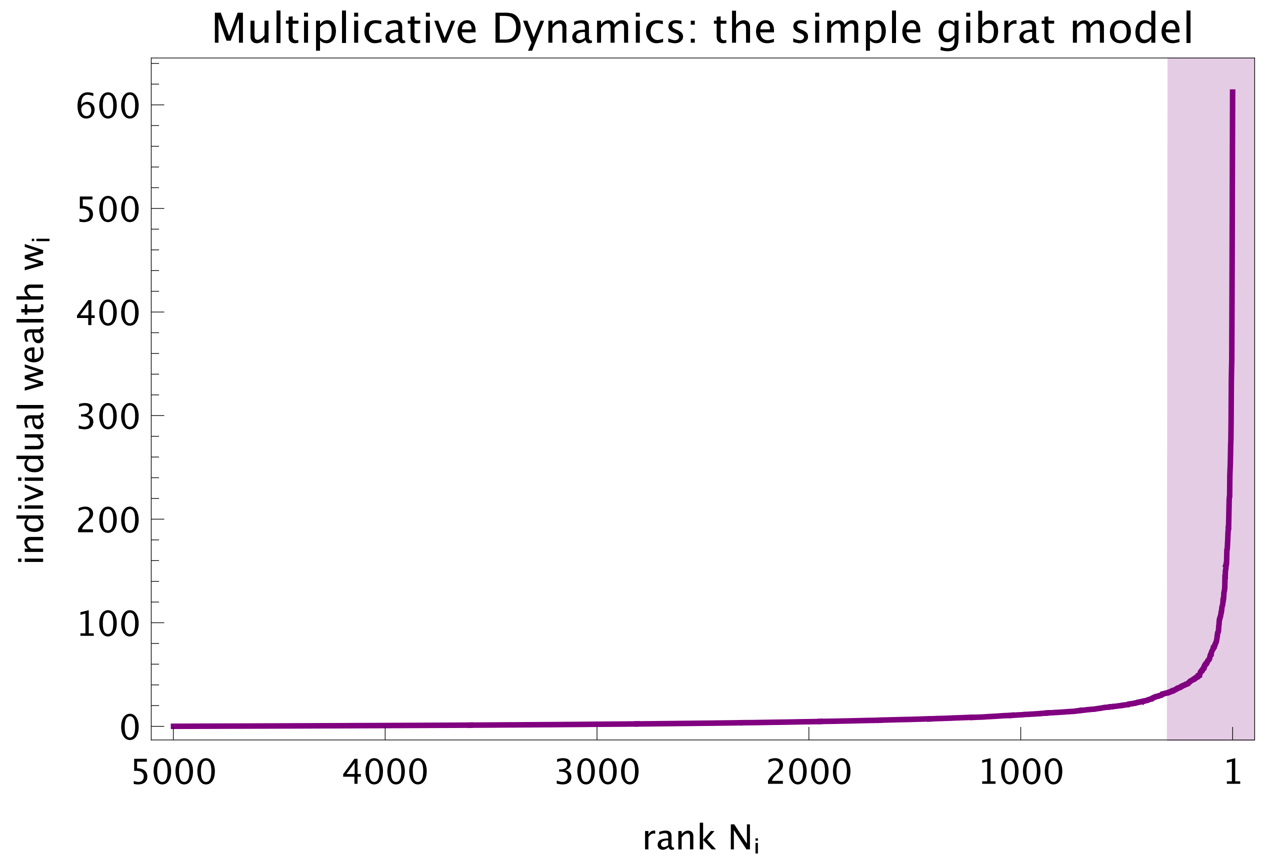

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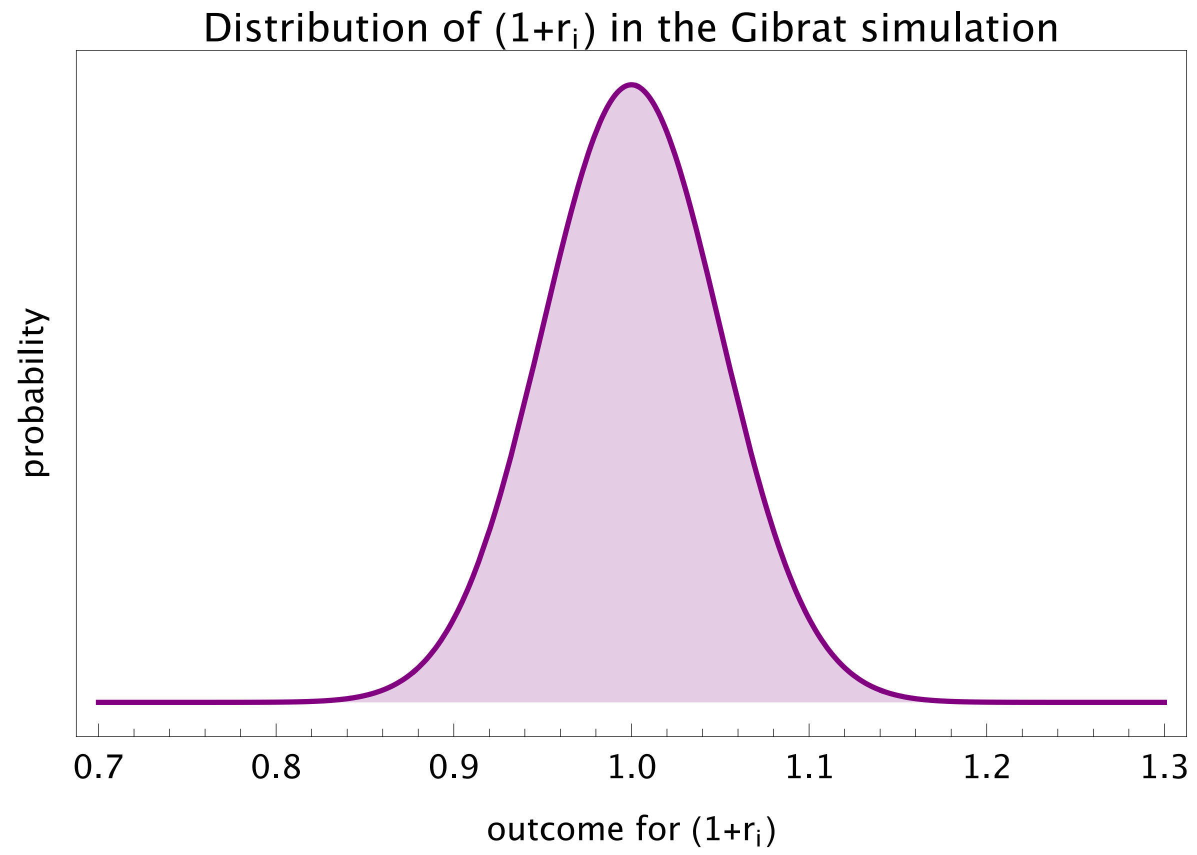

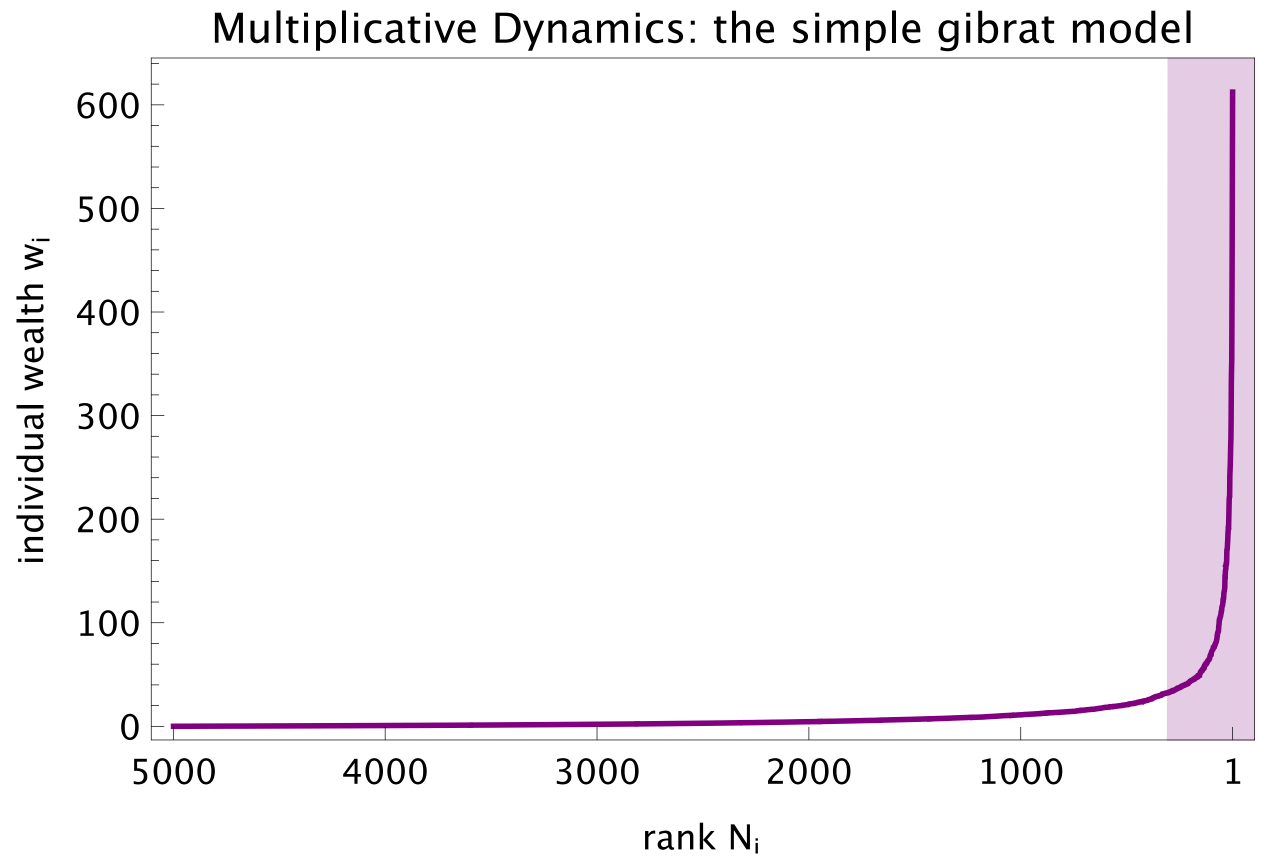

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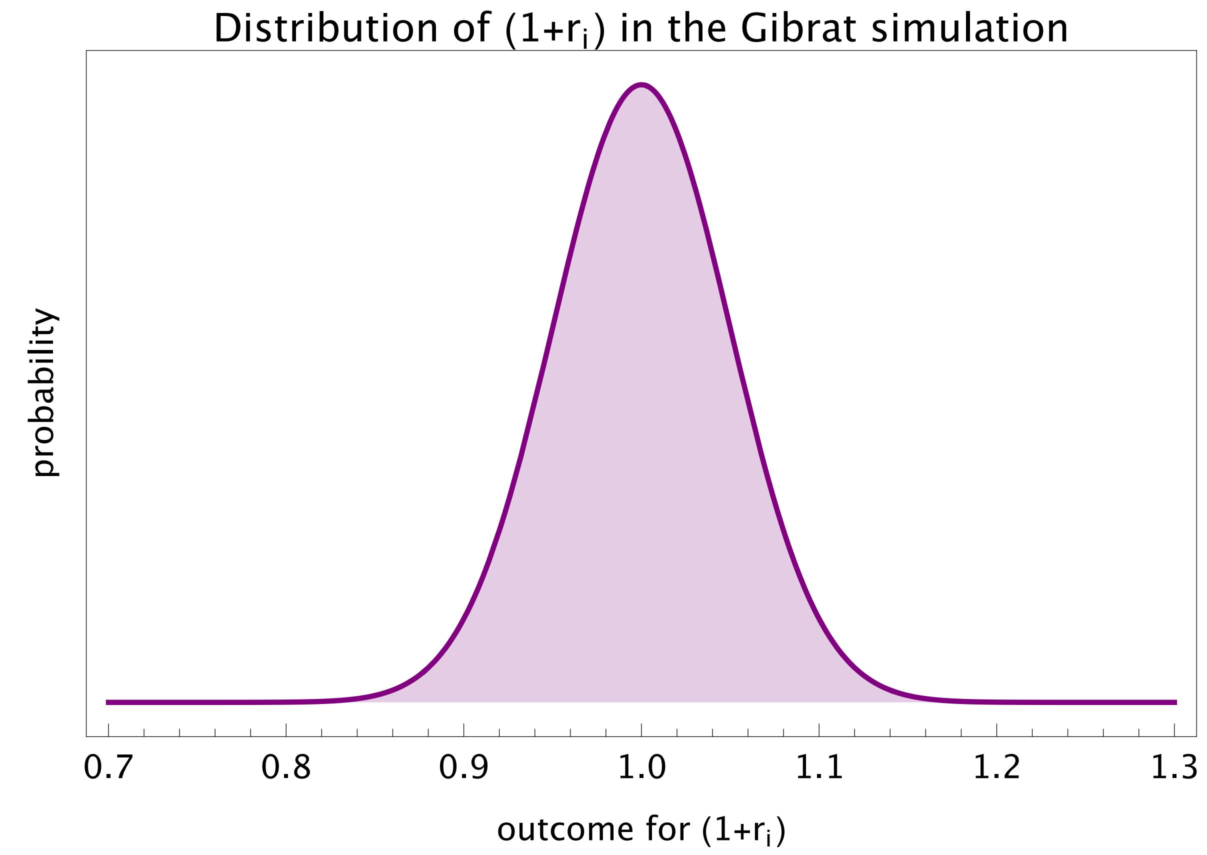

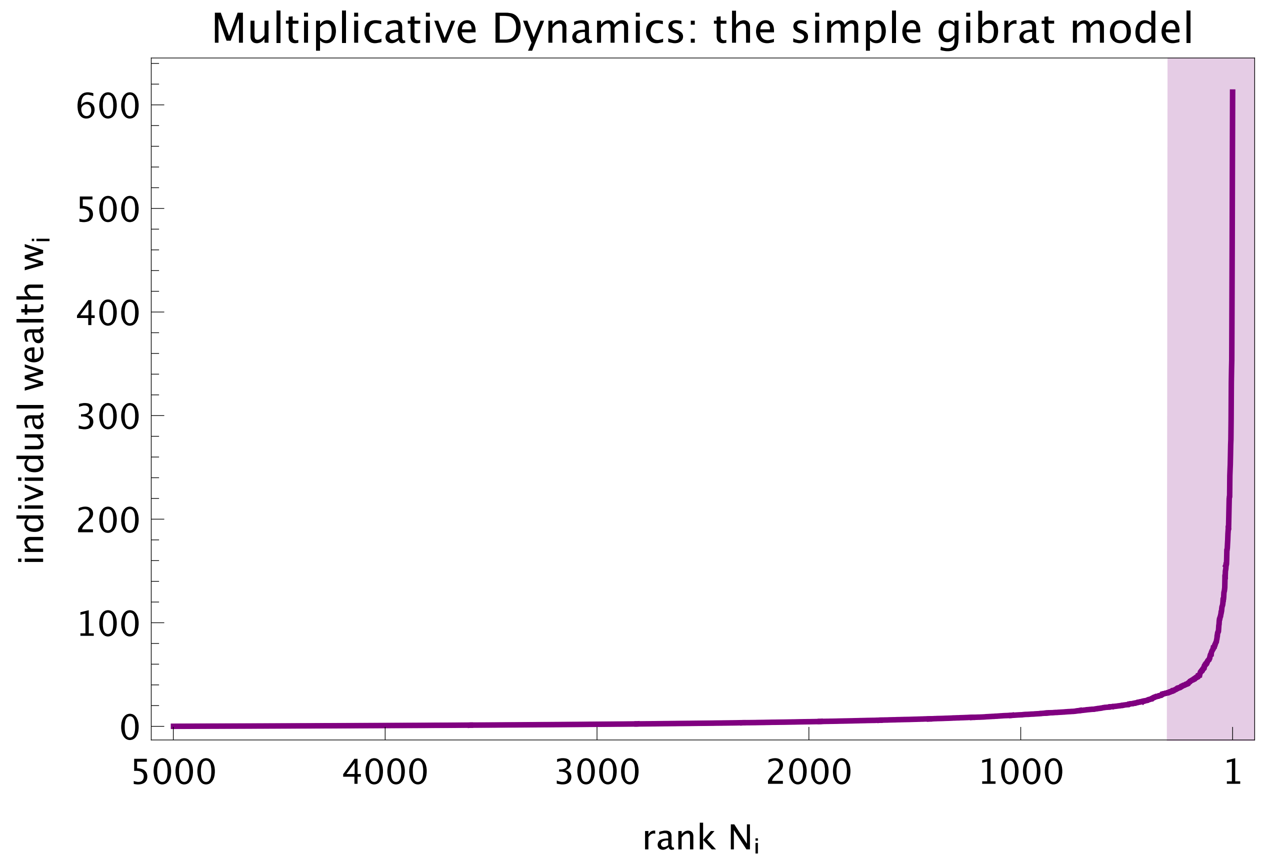

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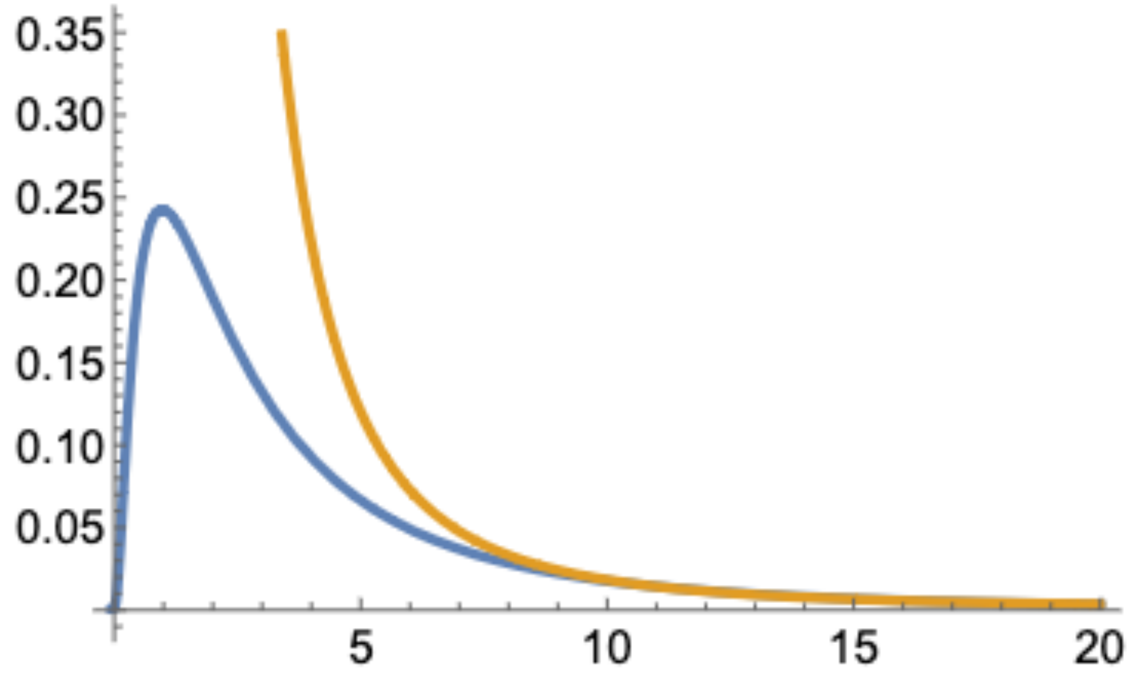

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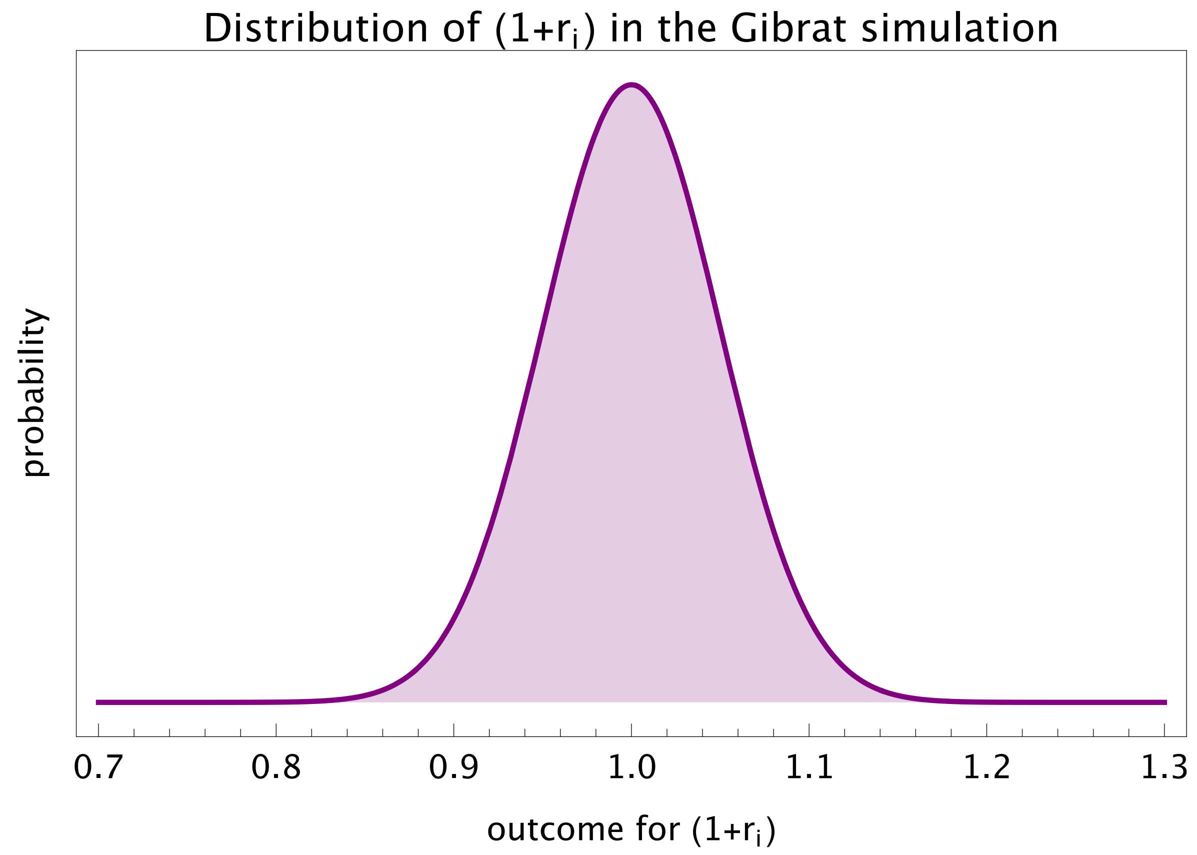

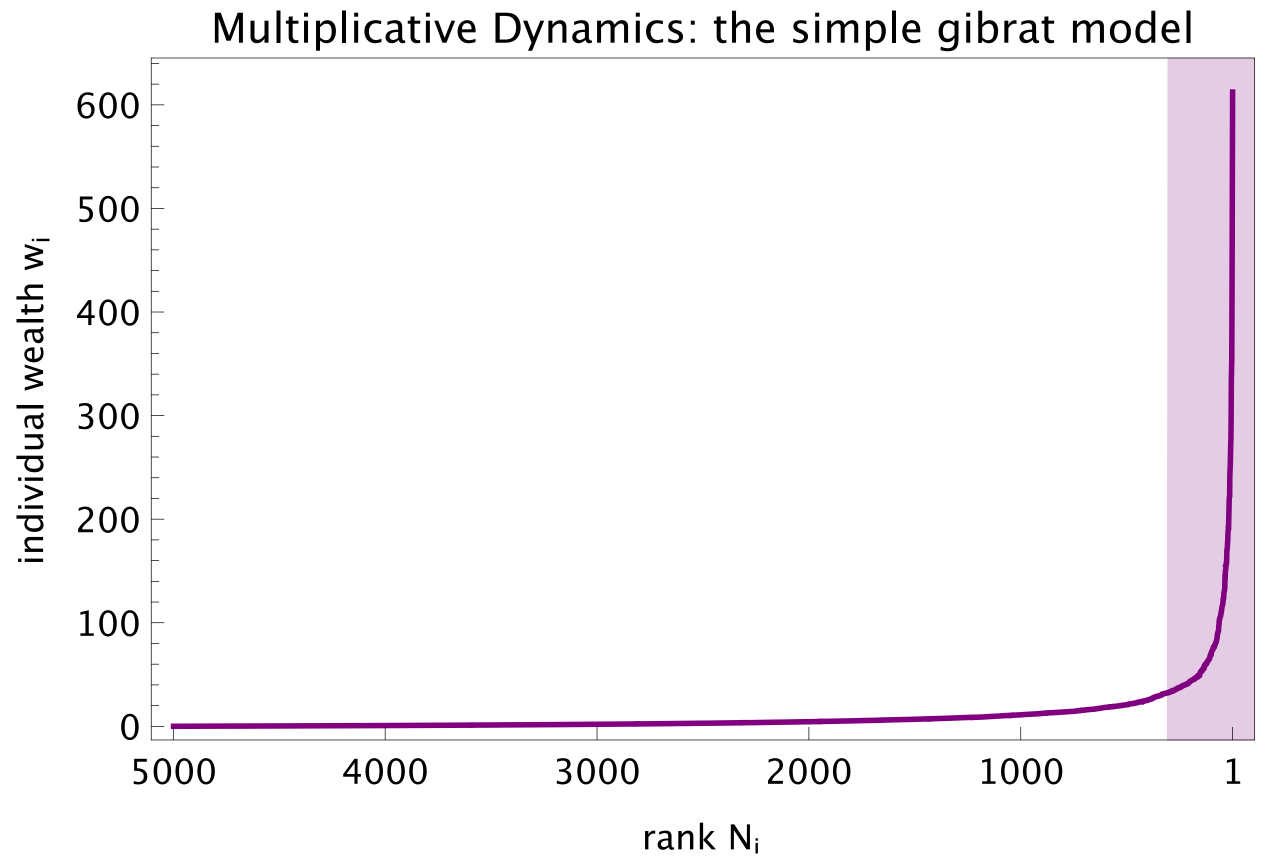

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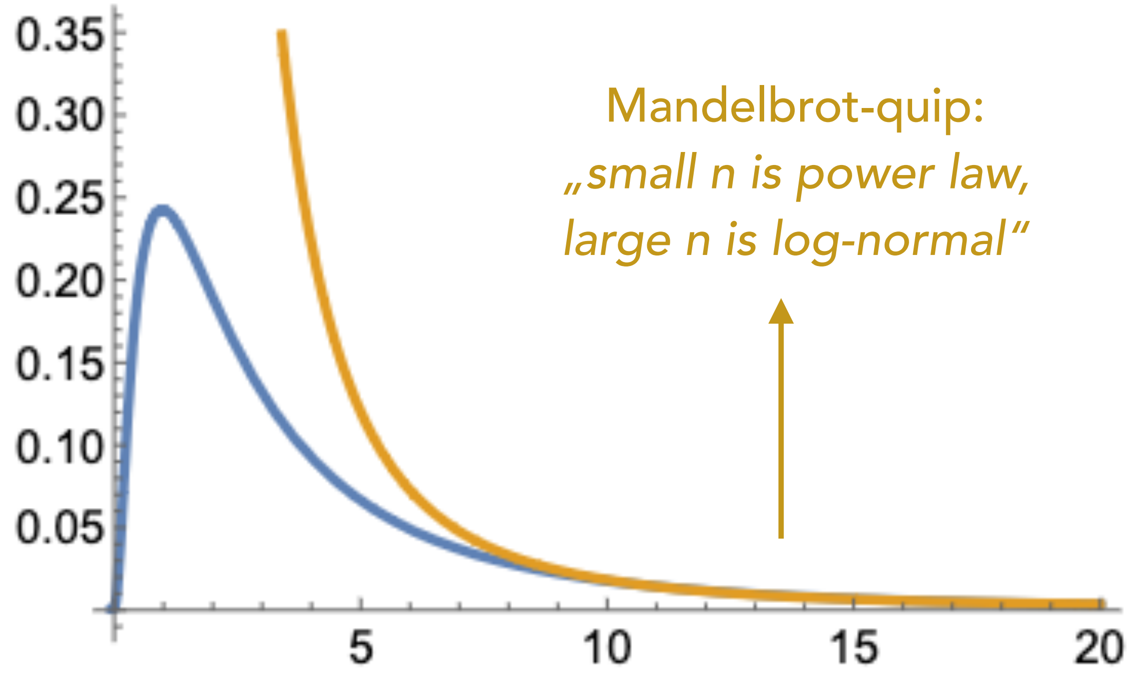

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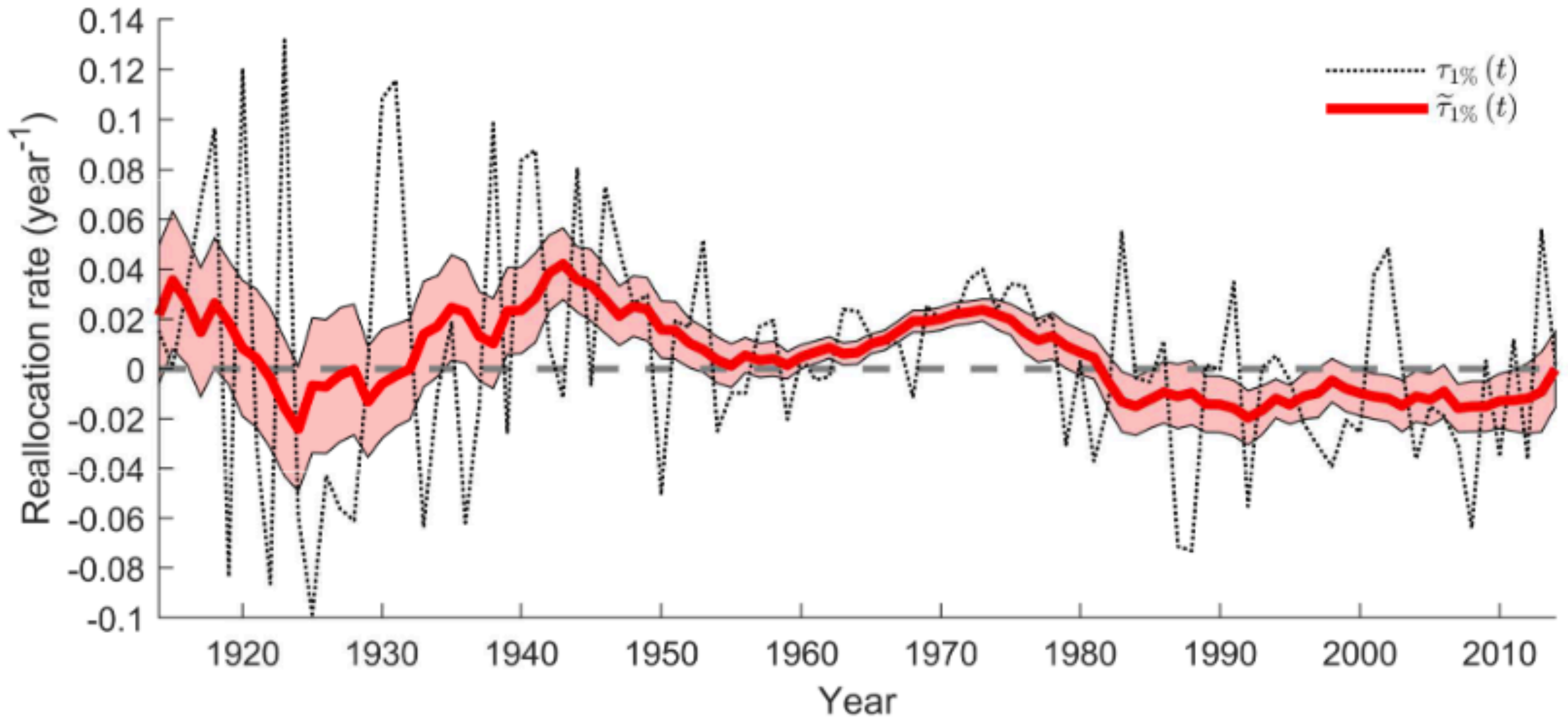

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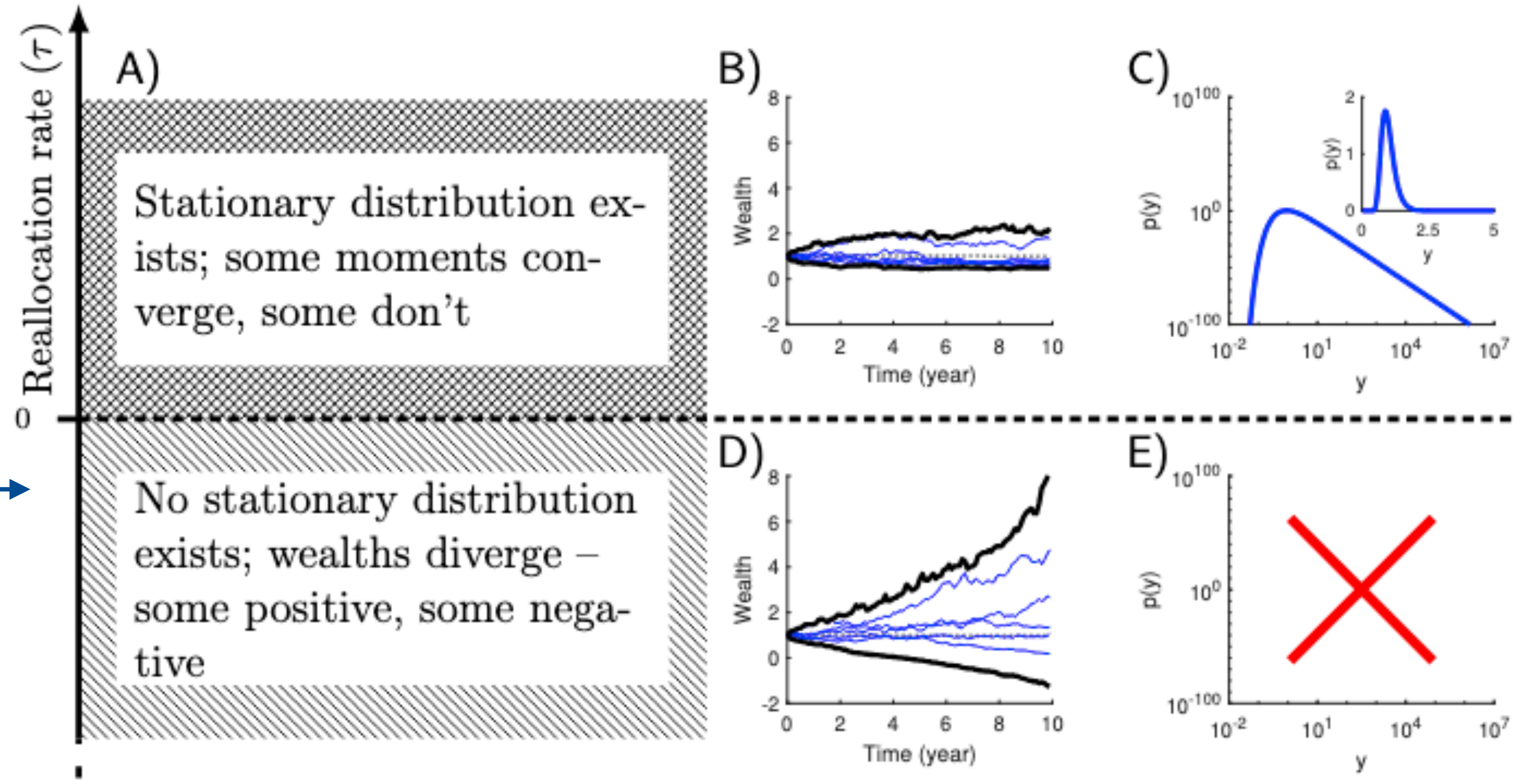

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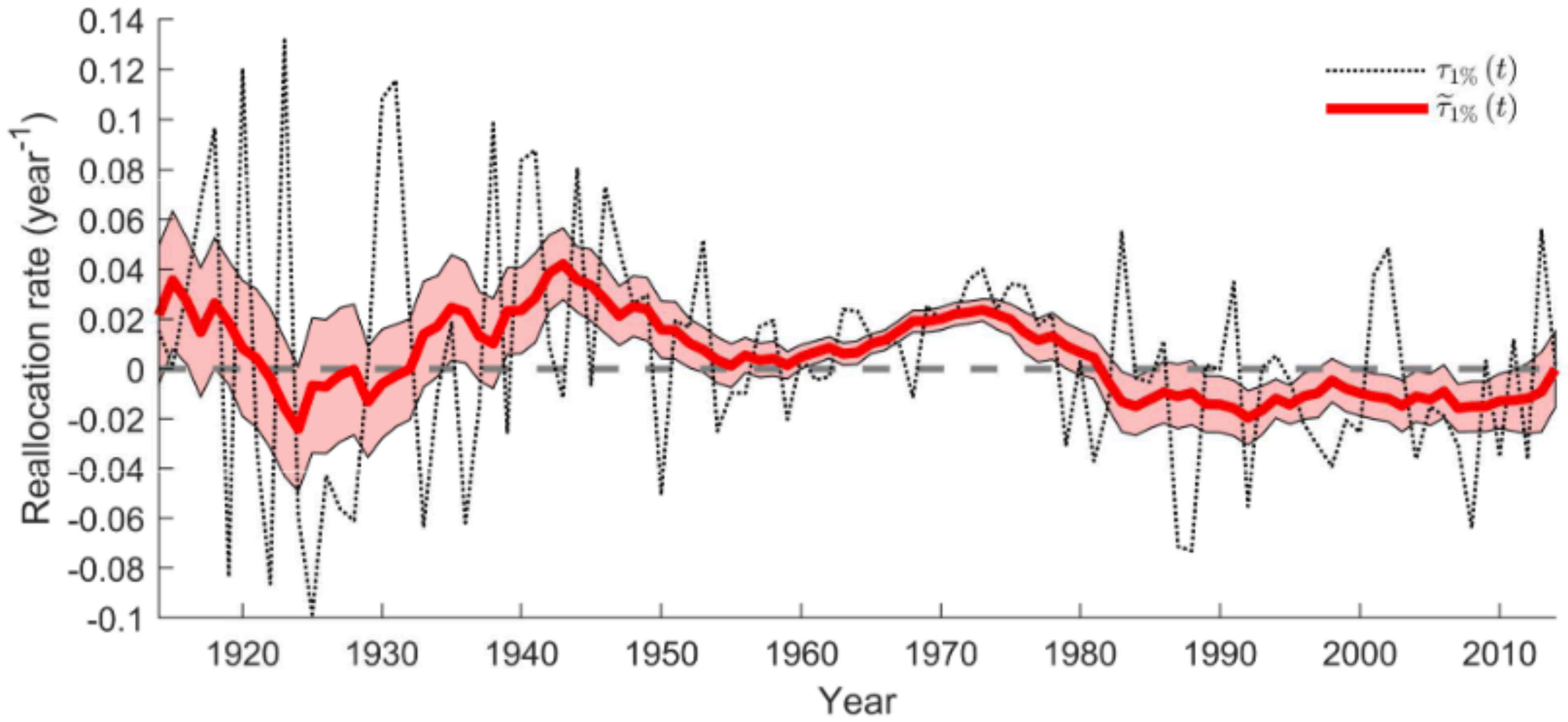

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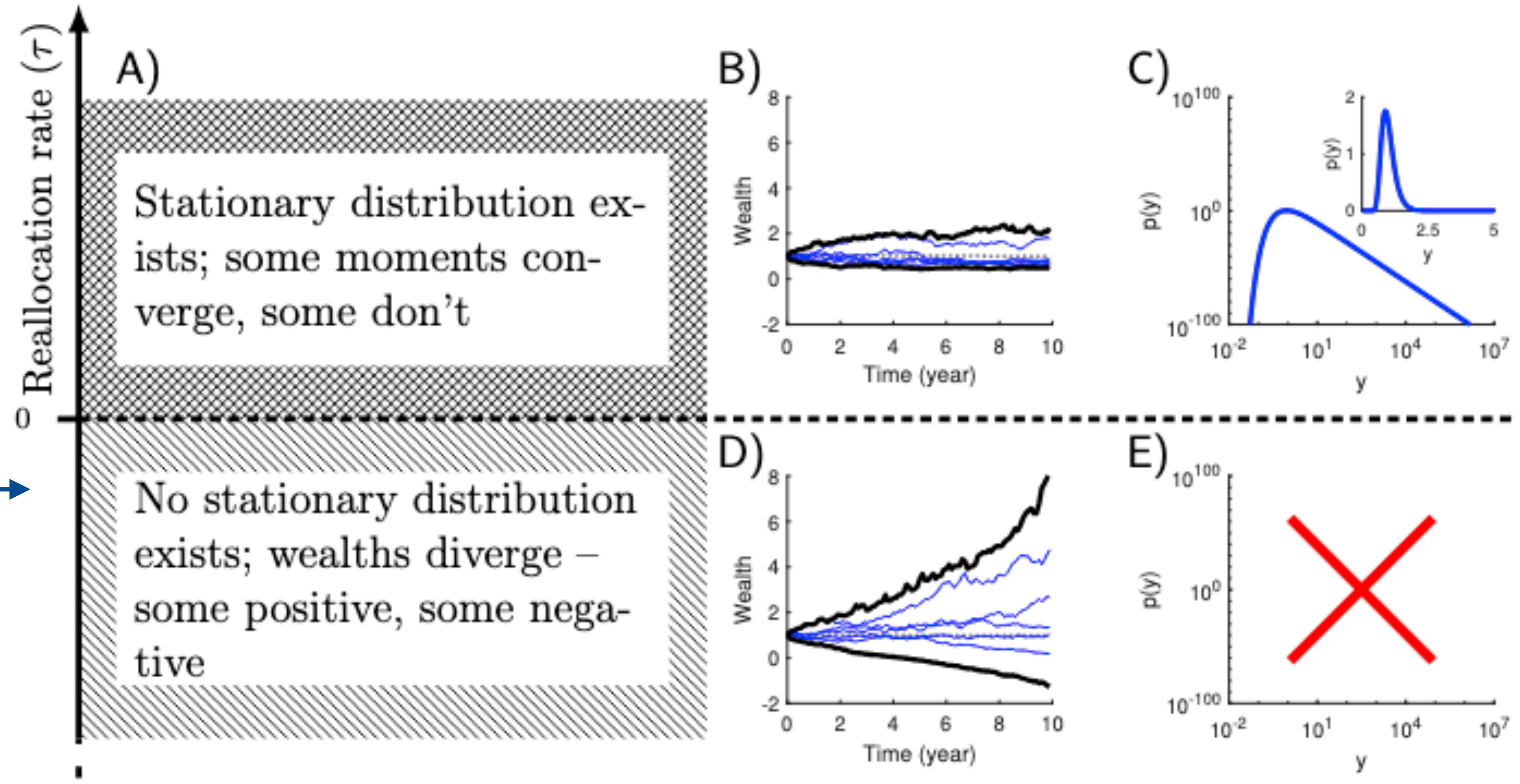

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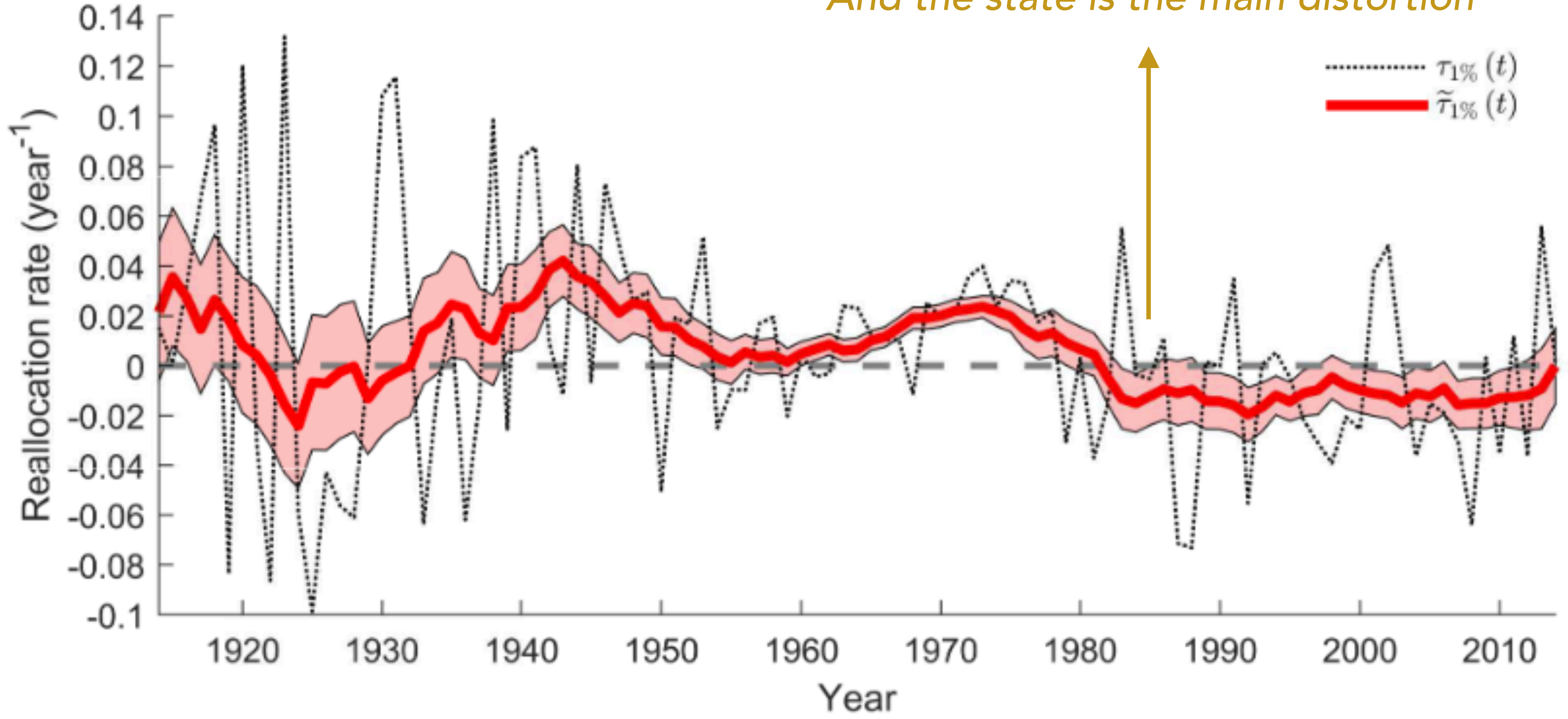
Econophysics abstraction:
„hey, the null-model of the world is multiplicative growth And the state is the main distortion"



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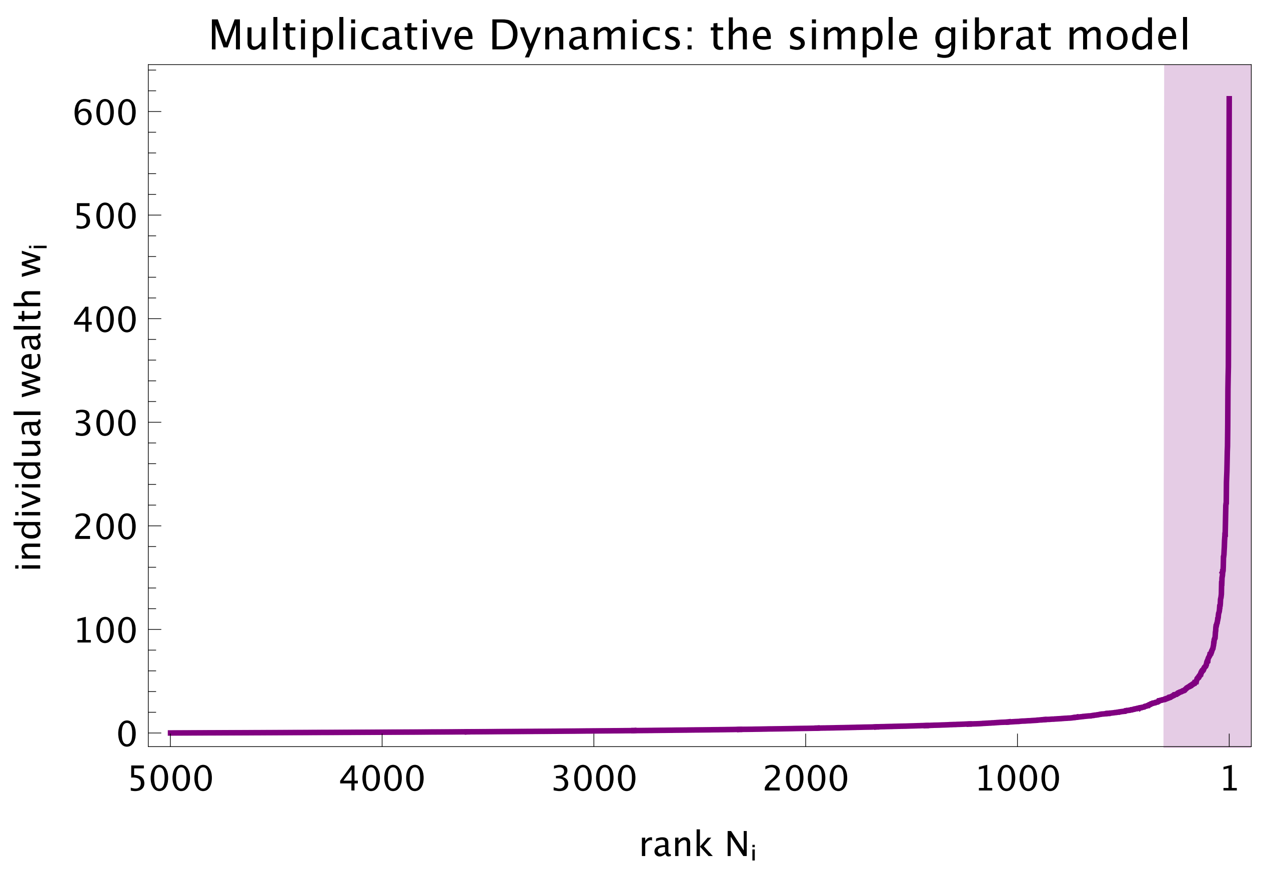
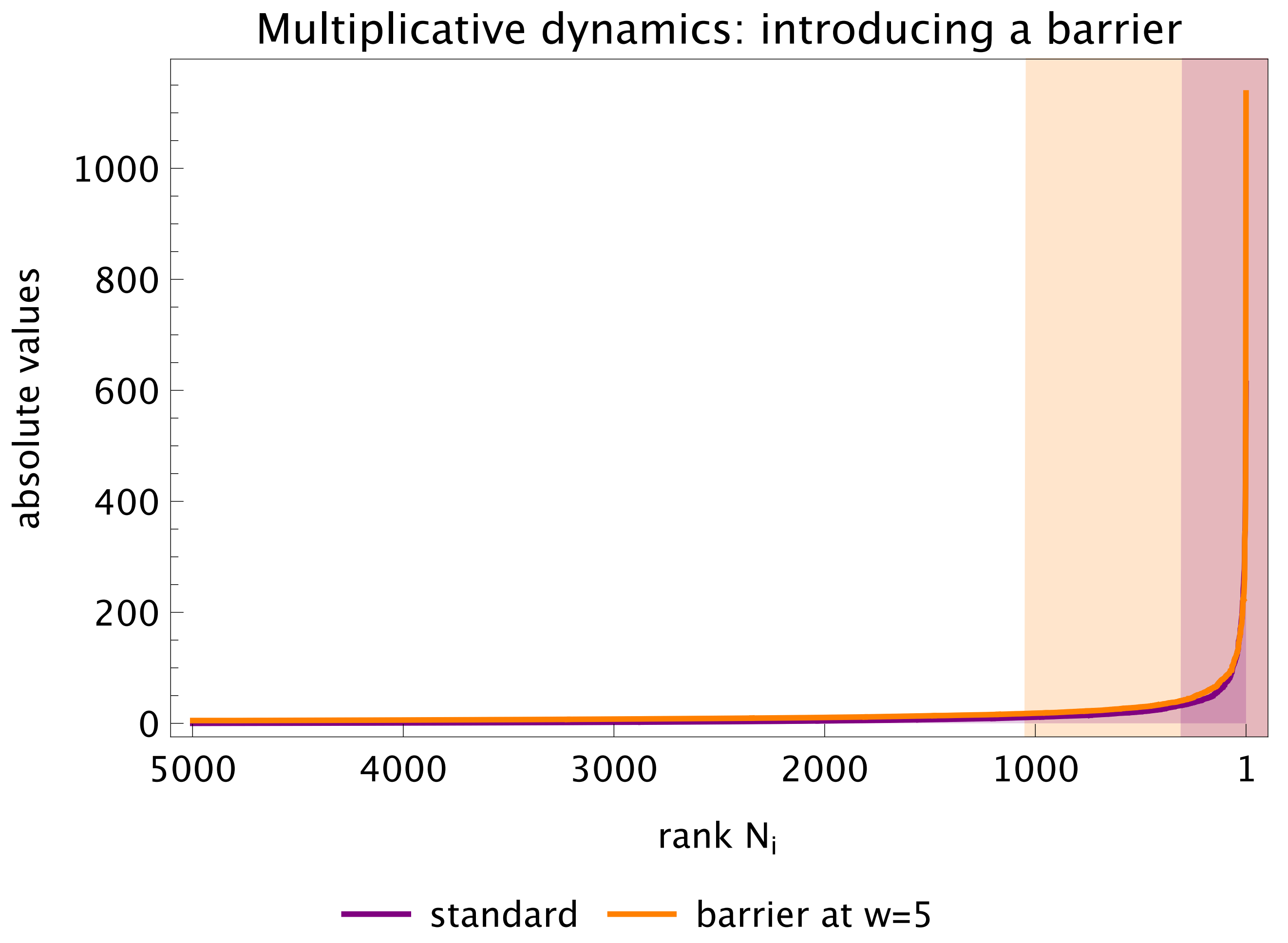
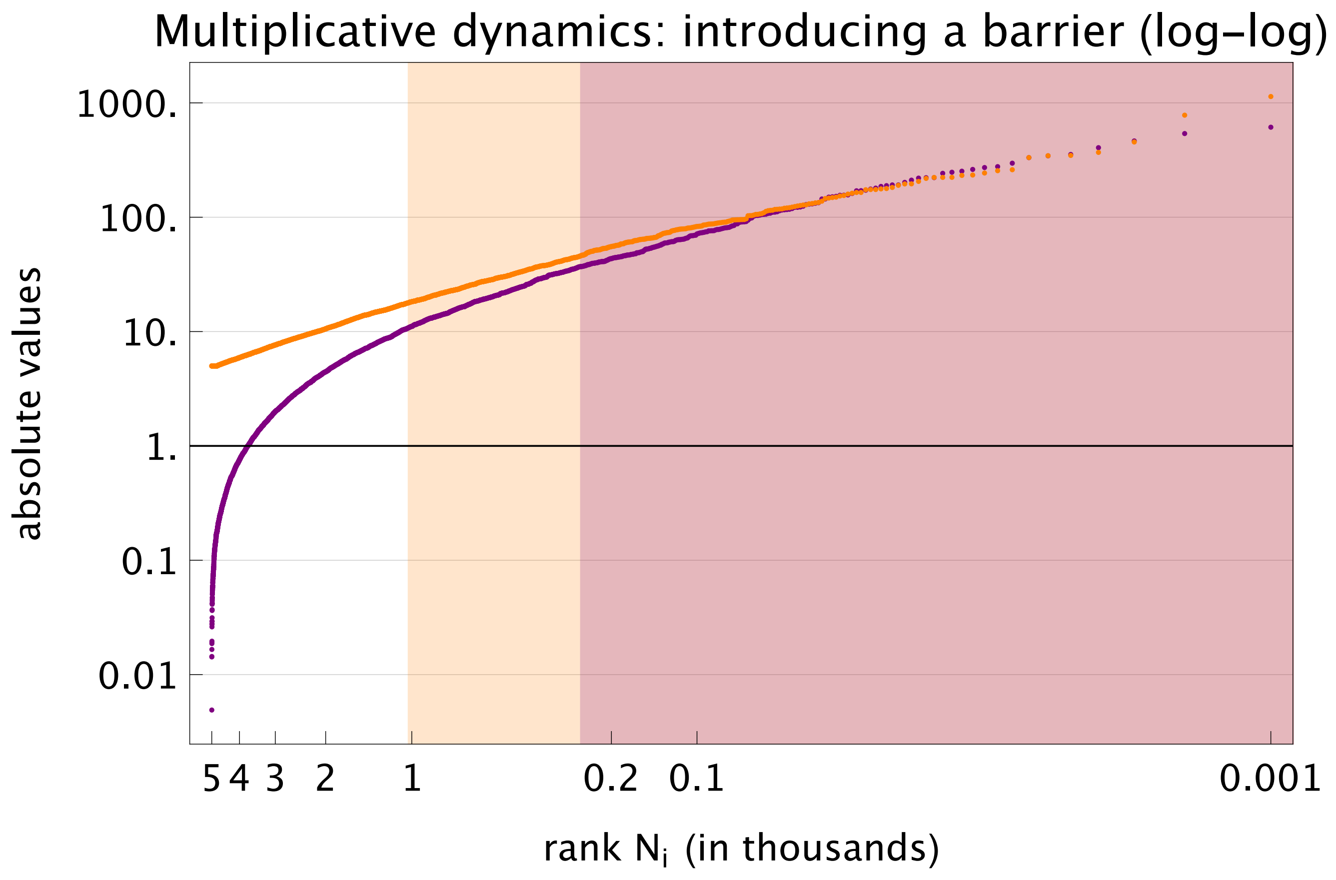
  - Simpler formulation, same **explosive** properties







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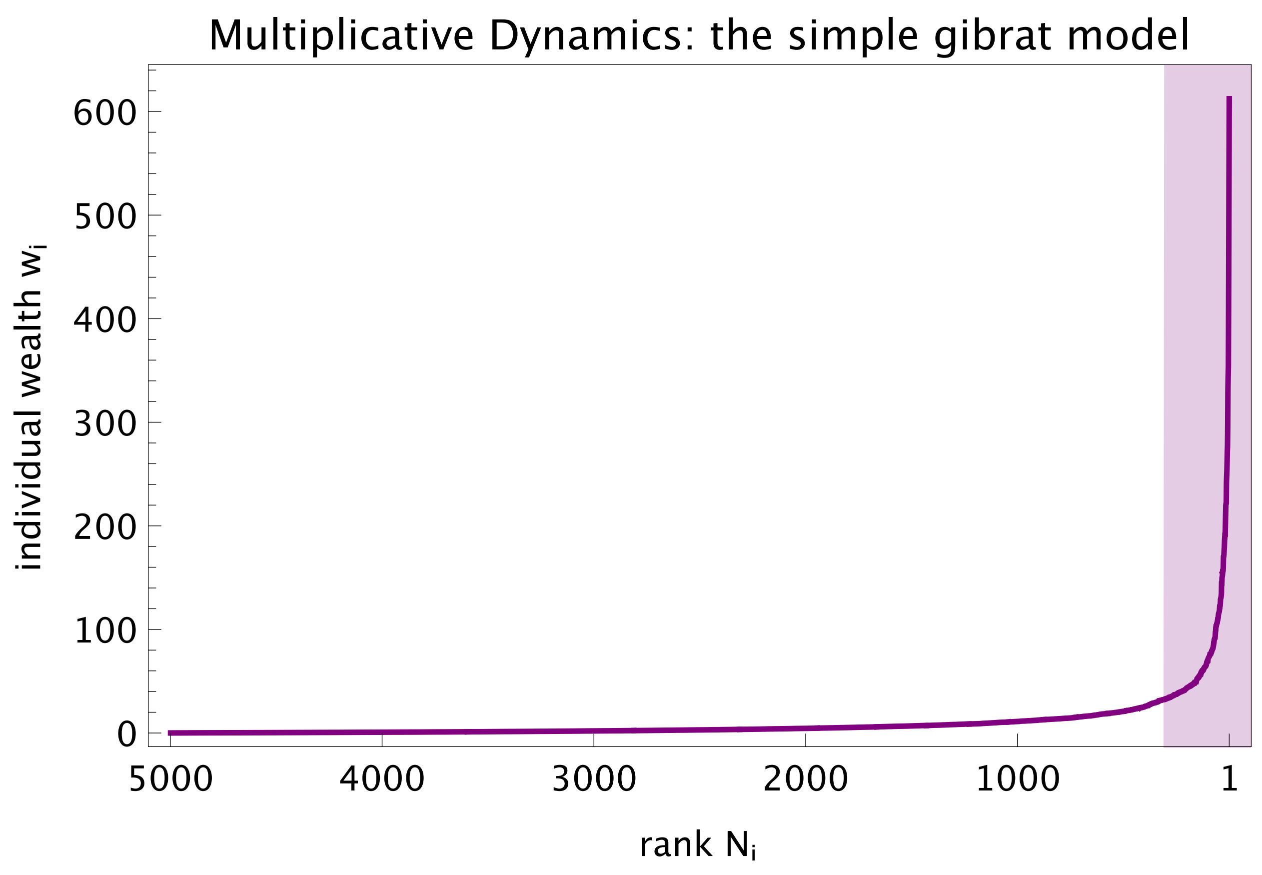
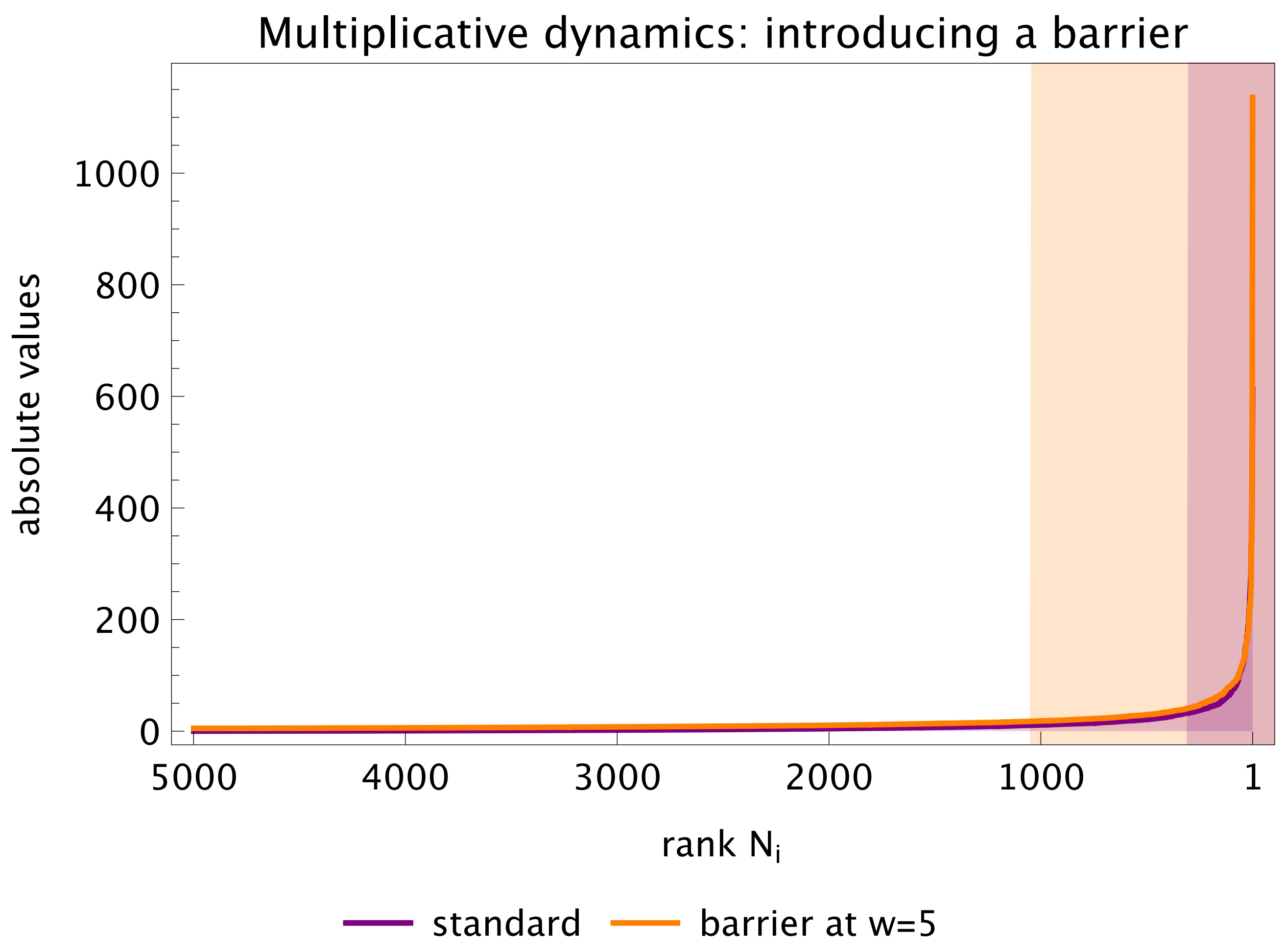
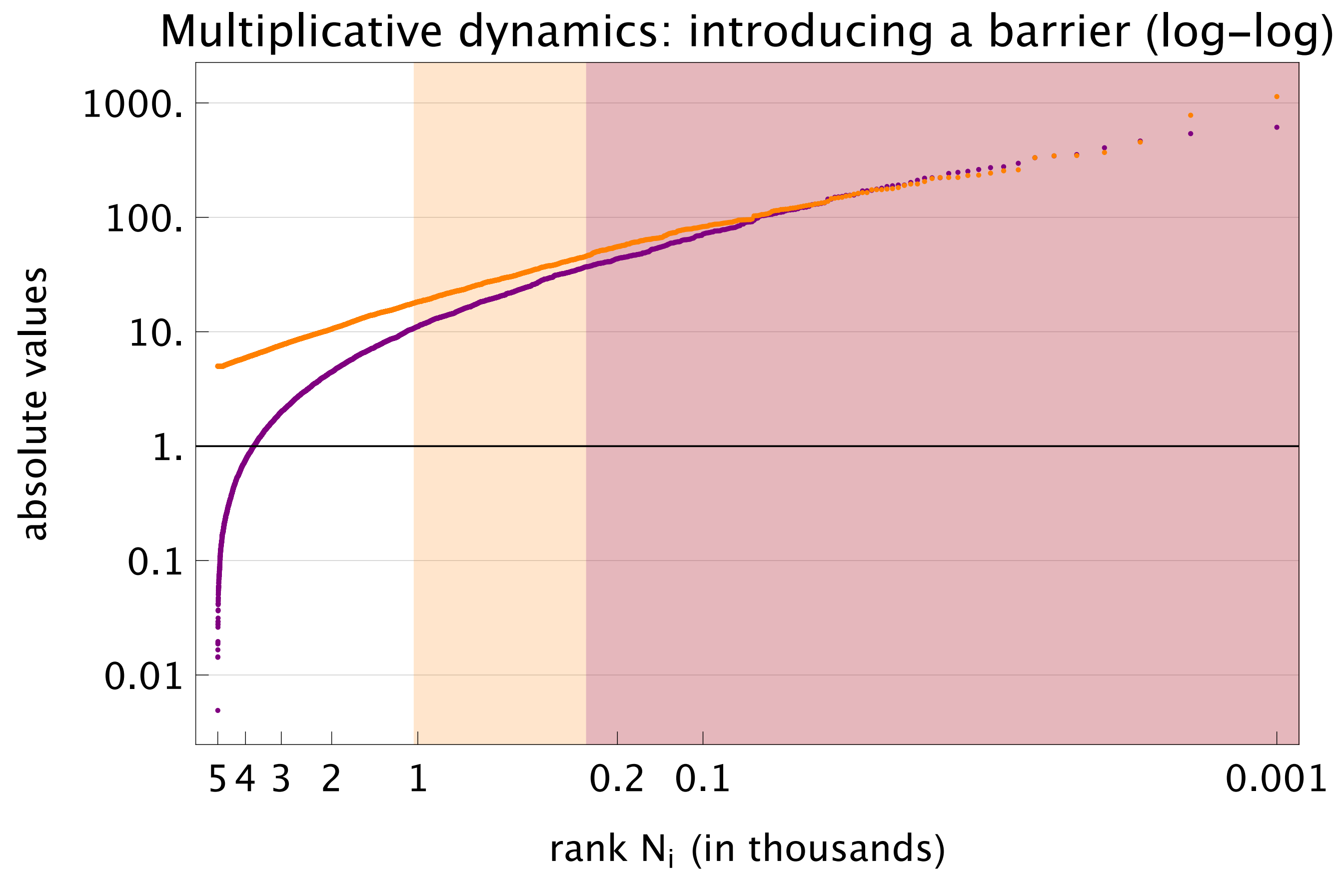




- **Another preliminary approach to modeling the welfare state**
  - Simply assume a basic safety net – **people cannot fall below some wealth w_{min}**
  - This works, resulting system is more equal.
  - However, agents do not drop out of multiplicative regime – drives system towards a power law distribution more strongly; even **more explosive** (in one direction).
  - With this come, e.g., **higher maximal values** on average in repeated applications – which would change, if we apply some tax to finance preserving the minima.



Random multiplicative growth: Empirical perspectives

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Subsistence	Hunter-gatherer	α	0.46	0.39	0.15		
		β	0.16 ± 0.06	0.23 ± 0.11	0.17 ± 0.011	0.19 ± 0.05	0.25 ± 0.04
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	Horticultural	α	0.53	0.26	0.21		
		β	0.17 ± 0.05	0.26 ± 0.11	0.09 ± 0.09	0.18 ± 0.04	0.27 ± 0.03
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Surplus	Pastoral	α	0.26	0.14	0.61		
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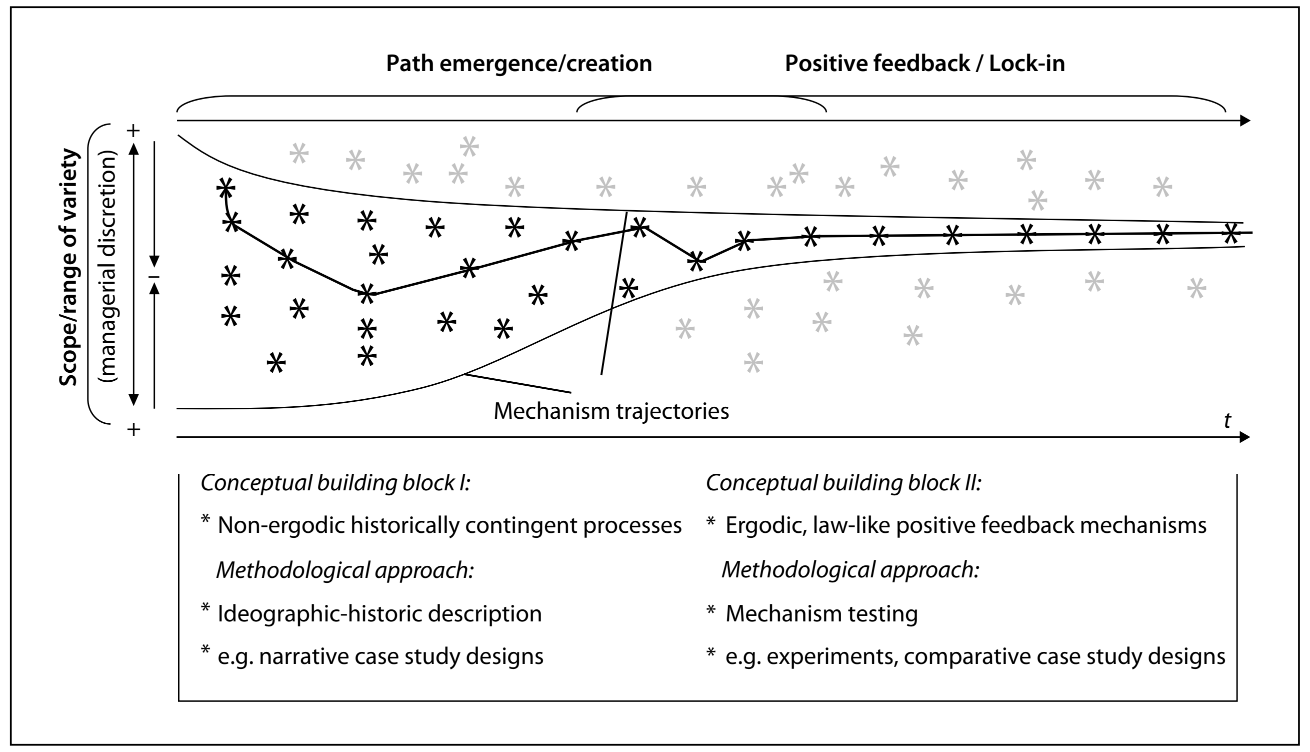
Figure 2 : Conceptual Building Blocks and Methodological Approaches to Path Dependence Research



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- Is a useful **null model**…
  - e.g. for empirically checking stability properties of distributions in economic history
  - Archetypical model for how complex patterns may emerge out of a simple setup.

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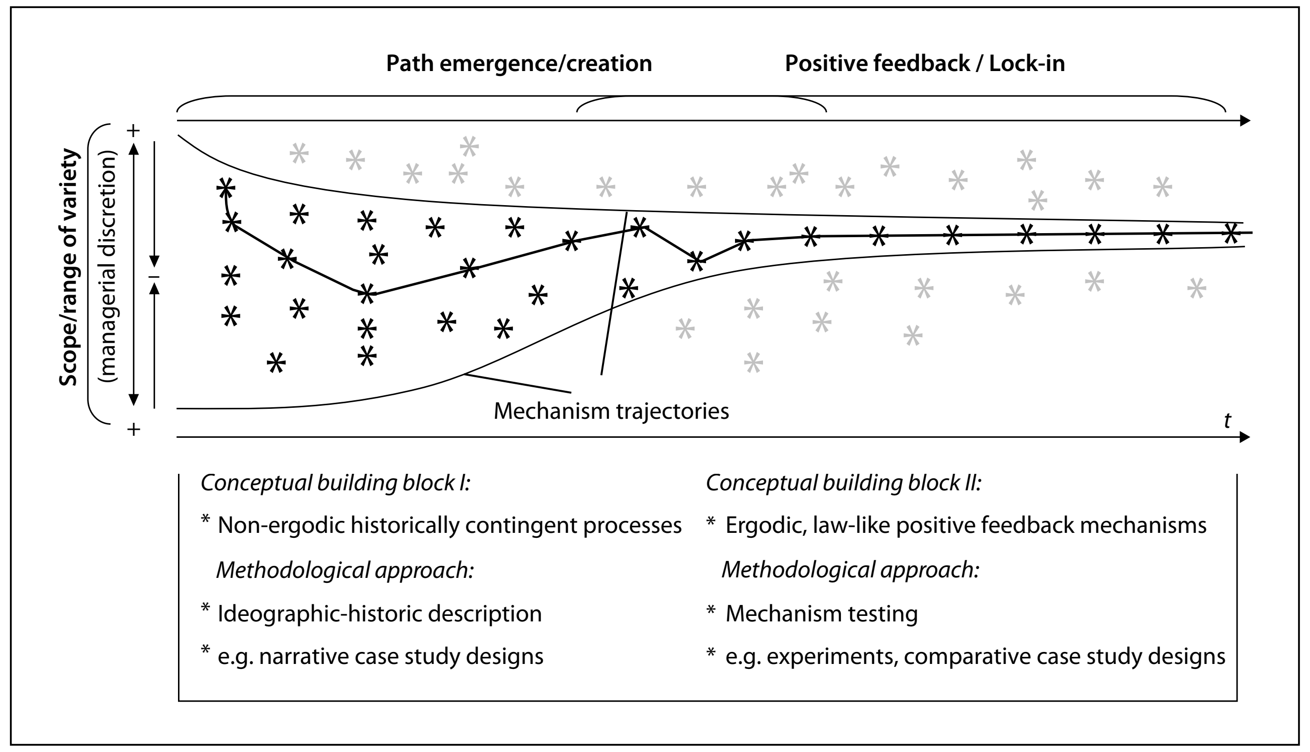

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Other interesting candidates

Other interesting variants for generating power laws

Time is probably running out ;-)

- Many very smart people say a Yule process is actually the same as the other cumulative advantage stuff
  - Models a birth process: several populations start with $n = 1$ and everybody can reproduce with some p – early mover advantage creates path-dependency – brings forth a Yule-Simon distribution, which is close to power law (under some…)
  - Somewhat in-between: driven by randomness, but cumulative advantage kicks in endogenously
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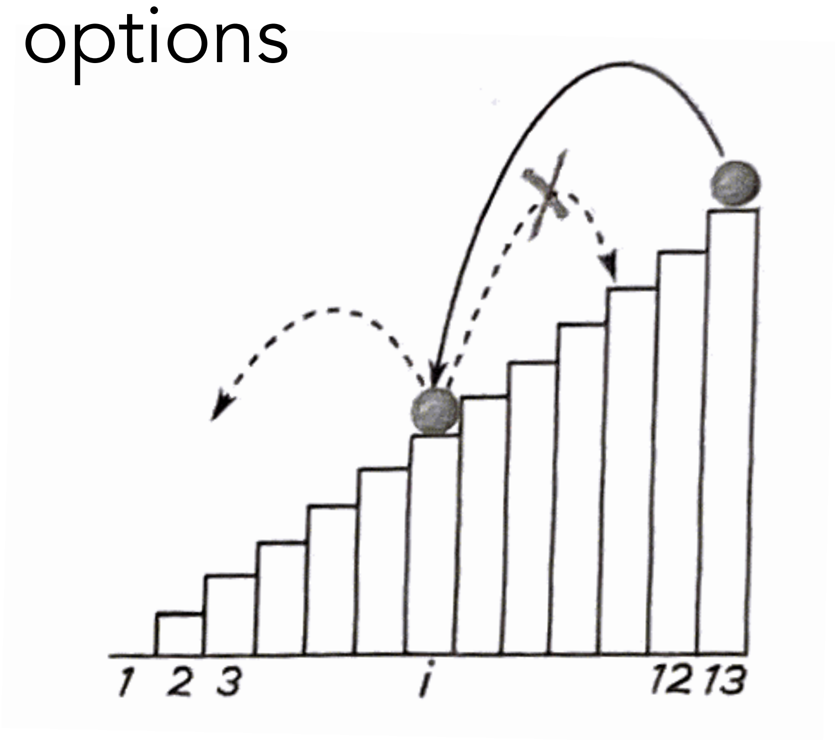

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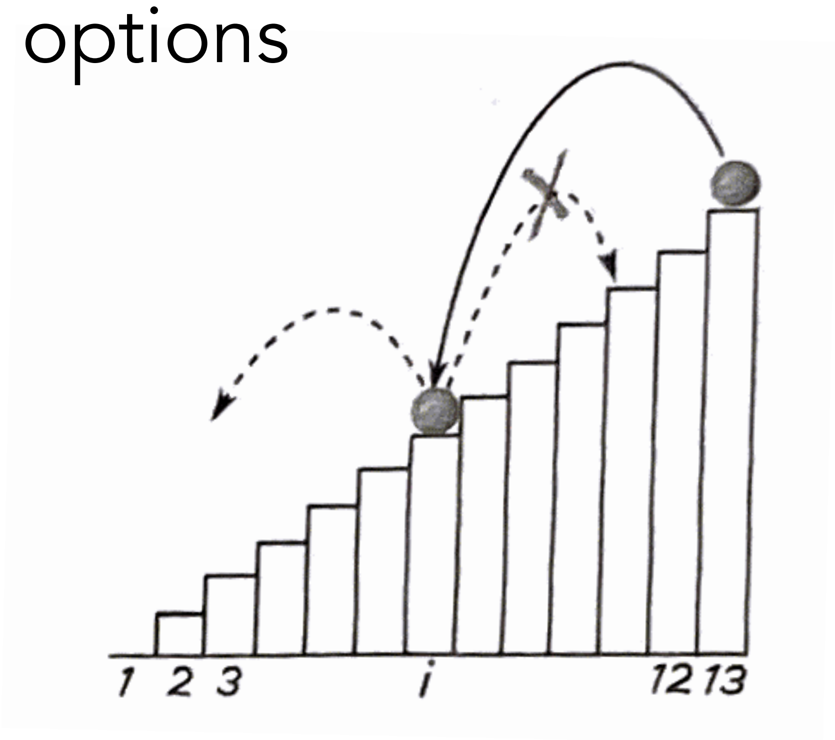

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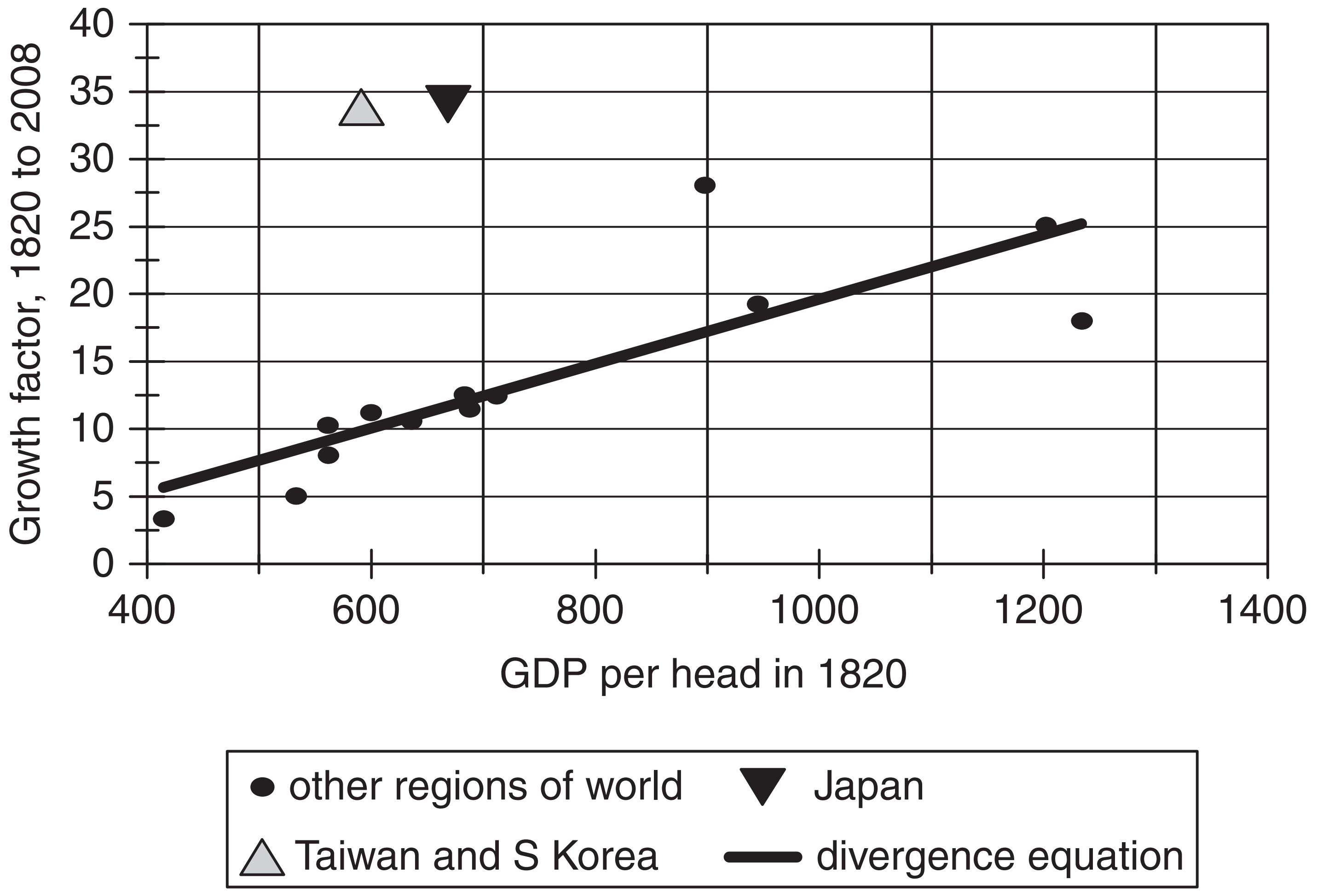

Allen, Robert C. (2017): The Industrial Revolution. OUP.

Conclusion

- Power laws are somewhat ubiquitous in economics
  - Power laws appear often in empirical data – unclear, whether / to extent this is driven by one or various mechanisms
  - Power laws resonate with key interests of heterodox economists / political economists: distribution, emergence, power, stratification, (in)stability etc.
  - Power laws are understudied (or at least they are understudied from a pol econ – perspective?).
- Simple models for generating power laws can be fun ;-)
  - Are nice to represent core intuitions, but should be taken with two grains of salt, at least.
  - Nonetheless, we found that more complicated models (like RGBM) can be traced back to these simpler ones.
  - Helpful also for checking, which intuitions hold formally and in what way (e.g. log-normal vs. power law).

Many thanks for your attention!

Backups

Some explanations / proofs

2. Universality

For large values of n, the model has a behavior that is independent of X: indeed, for any fixed X we can determine the numbers

$$\mu = \mathbb{E}\log(1+X) \qquad \text{and} \qquad \sigma^2 = \mathbb{V}\log(1+X)$$

and can replace X by the Gaussian $\mathcal{N}(\mu, \sigma^2)$ to get the same qualitative behavior.

Proof. We can write

$$x_n = x_0 \prod_{k=0}^{n-1}(1+X_k) = x_0 \exp\left(\log\left(\prod_{k=0}^{n-1}(1+X_k)\right)\right)$$
$$= x_0 \exp\left(\sum_{k=0}^{n-1}\log(1+X_k)\right)$$

and for this inner sum we have the law of large numbers telling us that

$$\sum_{k=0}^{n-1}\log(1+X_k) \sim \mathcal{N}(n\mu, n\sigma^2)$$

which is independent of X and depends only on mean and variance of $\log(1+X)$. □